

# Stock market efficiency beyond the joint hypothesis: Reevaluating the Moroccan stock market Efficiency through ARFIMA-FIGARCH model



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**Abstract** In the context of an unstable international environment, the assessment of the state of stock market efficiency remains of paramount importance. For this reason, we intend to implement ARFIMA-FIGARCH modeling over the period from 2 January 2013 to 29 December 2023. This will enable us to reevaluate efficiency beyond the problem of its joint hypothesis in the context of the Moroccan stock market. The results of our study indicate the presence of substantial evidence supporting the existence of long memory in both returns and volatility of the Moroccan stock market. These findings call into question the foundations of the Efficient Market Hypothesis beyond the Joint hypothesis problem, while offering novel insights into the dynamics of an emerging market. The findings have a practical significance in the fields of forecasting and risk management, as the persistence of return and volatility dynamics directly affects investment strategies and policymaking. Methodologically, the employment of a rigorous ARFIMA-FIGARCH framework enhances the robustness of the analysis by accounting for non-normality and heteroskedasticity in daily financial data.

**Keywords:** market efficiency, joint hypothesis, Moroccan stock market, ARFIMA-FIGARCH

## 1. Introduction

In the global economy, stock markets play a pivotal role, serving as a dynamic component of the economic landscape. Fundamentally, stock markets facilitate the mobilization of capital by entities engaged in production through the issuance of financial instruments. The collective expectations of investors regarding the economic and health futures of companies serve to transform the instruments traded on stock markets into barometers, that accurately reflect the dynamics of wealth creation. This occurs through the operation of supply and demand mechanisms.

Indeed, the ideal state of a stock market is one in which prices provide accurate signals regarding the allocation of capital (Nik-Khah, 2023). In other words, it is a market in which companies can adopt effective strategies for the utilization of financial resources. Consequently, investors may select among the securities that offer the most favorable returns at their own convenience. This assumption is based on the premise that security prices are fully reflective of all available information at all given times. This is precisely the description of an efficient market as described by Fama (1965).

Furthermore, for several decades, there has been significant interest among investors, companies, and regulators in understanding whether financial markets are efficient or inefficient. According to Fama (1965), a market is regarded as efficient if prices in that market reflect all available information, making it impossible for any market participant to outperform the market at any given moment. Moreover, the strength of this hypothesis is generally associated with the specific form of efficiency in question (Maiti, 2021). In terms of the type of information included; historical, public, and private information, Fama (1970) identifies three levels of efficiency, which he terms weak, semi-strong, and strong respectively. From an empirical standpoint, the joint hypothesis presents an inherent challenge in that it is difficult to disprove the market efficiency. This implies that the Efficient Market Hypothesis is a tautology (LeRoy, 1976).

In another point of view, Behavioral finance posits that investors' reactions are not rational (De Bondt & Thaler, 1985; De Bondt, 2020), and that their behavior toward information is excessive, even in the case of macroeconomic data (Shiller, 1981). This finding aligns with the principle of bounded rationality, as postulated by Herbert Simon (1957, 1982). According to this principle, the integration of information into financial prices is subject to delays due to the limited cognitive capacity of investors. These persistent mechanisms of biased assessments imply that past shocks slowly persist in future returns, subsequently giving rise to stock market anomalies.

In this regard, (Fama, 1970, 2014) recognized the necessity of employing a theoretical model of returns to test the Efficient Market Hypothesis, thereby acknowledging the problem posed by the joint hypothesis. This makes it challenging to ascertain whether the market is inefficient or whether the model employed is inadequate for the context under investigation. To remedy this difficulty related to the joint hypothesis problem, several empirical finance studies have attempted to apply advanced econometric memory models capable of taking into account the main characteristics of financial time series, particularly, ARFIMA (introduced by Granger & Joyeux (1980) and Hosking (1981), FIGARCH (Baillie et al., 1996) and other hybrid models (ARFIMA-FIGARCH). This is on the basis that the presence of long memory behavior in financial data automatically calls into question the Efficient Market Hypothesis, especially in the case of an empirical model offering an appropriate specification to the data mobilized.

In the context of developed financial markets, based on the estimation of ARFIMA models, CHRISTODOULOU-VOLOS et al. (2013) highlighted significant long-term dependencies in the returns of twelve European markets studied (Austria, Denmark, France, Finland, Germany, Greece, Netherlands, Italy, Norway, Spain and Turkey), calling into question their efficiency in the weak form. Similarly, Cavaliere et al. (2015) confirm the existence of fractional integration in commodity markets, by bootstrapping ARFIMA models. Nguyen et al. (2020) analyzed Hong Kong's Growth Enterprise Market, which is recognized as one of the world's top prosperous stock markets for SMEs. In this paper, the authors revealed a persistent inefficiency characterized by a double long memory in GEM returns and volatility, via ARFIMA-FIGARCH and FIAPARCH models.

From the point of view of emerging stock markets, Tripathy (2022) and Boubaker et al. (2022) respectively show that long memory is a dominant feature in BRICS and GCC indices, accompanied by a high degree of shock persistence in the volatility of the markets under study. In the same direction, Kasman et al. (2009) have already confirmed this behavior in Central and Eastern Europe (CEE), revealing a double long memory property in the majority of CEE markets. Contreras Reyes et al. (2024) reinforce the long memory result in returns dynamics by highlighting the predictive superiority of ARFIMA-GARCH models in the Chilean SSP context.

In the African context, Dum et al. (2023) identify a long memory in the monthly returns of the Nigerian All Share index, highlighting the superiority of the ARFIMA model over the standard ARIMA model. While on the Ghana Stock Exchange, Nkrumah Ababio et al. (2019) use the ARFIMA-FIGARCH model to reveal a structure of returns that is predictable, calling into question the efficiency of this market. Finally, other recent studies such as Eshun & Tweneboah (2025) and Basira et al. (2024) focusing respectively on the West African Monetary Zone and African commodity markets, confirm the presence of return and volatility dynamics characterized by long-memory dependencies.

In a similar line of research, Anoruo & Gil-Alana (2010) examined ten African markets, including Morocco, Egypt, Tunisia, Nigeria, and Kenya, employing the fractional integration approach. The findings indicated the presence of a memory process exclusively in the returns of the Egyptian and Nigerian indices, with a more modest manifestation in Kenya, Tunisia, and Morocco. In a recent study, Arewa et al. (2022) corroborated the existence of long memory behavior in the returns of the Nigerian and South African indices. In the Moroccan context, Omerani et al. (2022) demonstrated the presence of long memory in returns and specific calendar anomalies. To do so, they applied the ARFIMA model with standard GARCH extensions. However, the aforementioned research was constrained in its scope, as it exclusively evaluated the long memory in returns, while neglecting to consider the persistence of conditional volatility.

In terms of the 34th edition of the Global Financial Centers Index, the Moroccan stock market is positioned as the third most significant financial center in the Middle East and North Africa (MENA) region and the most significant in Africa. This ranking situates the Moroccan stock market in a strategic position within the African financial landscape. As a significant actor, it plays a pivotal role in the economic advancement of the Middle East and Africa region. The Moroccan stock market has established itself as a leading financial center, attracting both domestic and international investors in part due to a favorable regulatory environment, political stability, and remarkable economic dynamism. This important position in the region therefore raises questions about the efficiency of this stock market. On the other hand, there has been a notable absence of in-depth studies examining the efficiency of emerging markets in comparison with other global markets, most notably the Moroccan stock market.

Nevertheless, the Moroccan stock market, like many emerging markets, exhibits structural characteristics that differ from those examined in existing literature. These characteristics generally concern the market's state of low liquidity, weak connections to international financial markets, and limited investor diversification. These specificities make studying its efficiency all the more relevant. By modeling the behavior of returns and volatility, it is possible to decode the behavior of this market, which has been less explored in existing literature. In this context, returns and volatility behavior may show very slow information integration, justifying the importance of paying particular attention to the temporal dynamics of their time series. With regard to this, our study explores the possibility of a double long memory characteristic in returns and volatility, which calls into question the Efficient Market Hypothesis. Contrary to developed stock markets, where prices quickly and systematically integrate past and new information, emerging markets exhibit remarkable persistence of past shocks. This persistence, often ignored in the classical paradigm of efficiency, which constitutes a potential source of exploitable information for informed investors. Additionally, this research's principal methodological contribution lies in using ARFIMA models for returns and FIGARCH models for volatility, as well as referring to different probability distributions for more robust

statistical inference. This approach makes it possible to simultaneously capture long memory in the two key dimensions of financial time series with a rigorous methodology that addresses the joint hypothesis problem.

To this end, our study aims to evaluate the efficiency of the Moroccan stock market. In order to achieve this objective, we intend to employ the reasoning method of proof by contradiction. The initial hypothesis is that this market is totally efficient. The objective is to determine whether this hypothesis can be confirmed by using the available empirical data.

In order to achieve our objective, it is necessary to address the following central question:

*Beyond the problem of the joint hypothesis, to what extent can the ARFIMA FIGARCH model call into question the efficiency of the Moroccan stock market?*

In order to answer the central question, our research is structured into four main sections. Section 2 is devoted to an exposition of the data and methodology that was utilized in the study. The subsequent section, Section 3, presents and discusses the results, while Section 4 concludes the paper.

## 2. Materials and Methods

### 2.1. ARFIMA-FIGARCH modeling

The ARFIMA model, which was independently formulated by Granger & Joyeux (1980) and (Hosking, 1981), is commonly used to model long memory. In general, an ARFIMA (p, d, q) process is defined by the subsequent equation:

$$\varphi_p(L) (1 - L)^d Y_t = \theta_q(L) \varepsilon_t \quad (1)$$

Where  $Y_t$  is an integrated time series of order d. The autoregressive component, AR(p), is represented by  $\varphi_p(L)$ , while the residual-based moving average component, MA (q), is given by  $\theta_q(L)$ . In this context, the symbol  $L$  represents the lag operator, while the symbol d assumes fractional values within the interval [-0.5, 0.5]. The term  $\varepsilon_t$  represents the white noise process, which is assumed to be Gaussian.

The application of the ARFIMA model requires the absence of the ARCH effect, in other words, compliance with the assumption of error homoscedasticity, which has been widely refuted. It also requires the normality of residuals, which are specific characteristics of financial time series. Given the inherent volatility of financial series, which cannot be efficiently captured by a standard ARFIMA (p, d, q) model, it is recommended that volatility modelling be incorporated.

By posing;

$$\varepsilon_t = \sqrt{\sigma_t^2} z_t \quad (2)$$

Where;  $\sigma_t^2$  expresses the conditional variance of  $\varepsilon_t$ , while  $z_t$  represents a Gaussian stochastic process. In order to ensure stationarity, the parameters  $\alpha_i$  and  $\beta_i$  are to be estimated, subject to the following conditions:

$$\sum_{i=1}^q \alpha_i + \sum_{i=1}^p \beta_i < 1 \quad (3)$$

In response to the advancement of long memory processes, particularly as observed in ARFIMA model, Baillie et al. (1996) introduced the FIGARCH (fractionally integrated generalized autoregressive conditional heteroscedasticity) model to capture long memory of the studied serie's volatility. In contrast to the standard GARCH model, in which shocks are assumed to decay at an exponential rate, or the IGARCH model, in which shocks are assumed to persist indefinitely, the FIGARCH model represents a distinct approach. In this model, shocks are decaying at a slower hyperbolic rate (Ghalanos, 2020).

The standard GARCH equation (Bollerslev, 1986) can be presented as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \quad (4)$$

This model can be written in an alternative form as follows:

$$\sigma_t^2 = \omega + \alpha(L) \varepsilon_t^2 + \beta(L) \sigma_t^2 \quad (5)$$

In this context,  $L$  represents the lag operator, with  $\alpha(L) = \sum_{i=1}^q \alpha_i L^i$  and  $\beta(L) = \sum_{i=1}^p \beta_i L^i$ .

By introducing the variable  $v_t$ , defined by:

$$v_t = \varepsilon_t^2 - \sigma_t^2 \quad (6)$$

The standard GARCH model is written as an ARMA model:

$$[1 - \alpha(L) - \beta(L)] \varepsilon_{t-i}^2 = \omega + [1 - \beta(L)]v_t \quad (7)$$

Given that:  $\varphi(L) = \sum_{i=1}^{m-1} \varphi_i L^i$  where  $m = \max \{p, q\}$ .

The equation (7) is expressed in the following form:

$$\varphi(L) (1 - L) \varepsilon_{t-i}^2 = \omega + [1 - \beta(L)]v_t \quad (8)$$

Finally, the FIGARCH model (Baillie et al., 1996) is defined as follows:

$$\varphi(L) (1 - L)^D \varepsilon_{t-i}^2 = \omega + [1 - \beta(L)]v_t \tag{9}$$

In the context of the fractional differentiation parameter of the FIGARCH model, the value of D is bound between 0 and 1 ( $0 \leq D \leq 1$ ). The latter offers greater flexibility for modeling conditional variance, in that it incorporates the standard GARCH model as a special case when D is null ( $D = 0$ ) and the IGARCH model as a special case when D is equal to unity ( $D = 1$ ) (Kang & Yoon, 2013).

2.2. Data and methodology

The objective of this research is to re-examine the efficiency of the Moroccan stock market. To achieve this objective, we focus on the closing prices of the main MASI index, grouping all stocks listed on the Moroccan stock market. This index is calculated by a free-float market capitalization-weighted methodology, which is based on the concept of effective availability of shares for trading. Additionally, a capping factor is implemented, which imposes a limitation on the weight of any individual company to 20% of the MASI's free-float capitalization. This adjustment is made to ensure that the MASI index is providing an accurate representation of the reality of the Moroccan stock market.

This database, extracted from the official investing.com website, covers the period from January 2, 2013 to December 29, 2023.

In the course of our research, we will conduct our analyses on the basis of logarithmic daily yields, designated as  $R_t$ , calculated as follows:

$$R_t = \left[ \ln \left( \frac{C_t}{C_{t-1}} \right) \right] 100 \tag{10}$$

Where;

$R_t$  represents the logarithmic percentage return at time t.

$C_t$  represents the closing price on day t.

$C_{t-1}$  represents the closing price on day t-1.

In light of our research objective, we intend to adopt a post-positivist epistemological posture, employing a methodology based on a hypothetico-deductive approach. To address any potential tautology and avoid the problem of joint hypothesis, in our empirical study, we will utilize the principle of proof by contradiction.

Our study is thus based on the finding that there may be a long memory in the returns and volatility of the MASI index returns, which calls into question the efficiency of the Moroccan stock market.

In pursuit of this vision, our research involves testing the following hypotheses:

$H_0$  : The Moroccan stock market is efficient.

$H_1$ : The Moroccan stock market is inefficient;

$H_{1a}$  : The MASI index returns are characterized by long memory,

$H_{1b}$  : The volatility of MASI returns exhibits long-memory dependencies.

3. Results and Discussion

The descriptive statistics presented in Table 1 indicate that, on average, the logarithmic returns of the MASI index recorded a mean return of 0.004026 during the period under study. The standard deviation of 0.2343612 indicates a considerable degree of dispersion relative to the mean. This phenomenon is indicative of the volatile behavior exhibited by MASI returns, a characteristic that is consistent with the nature of financial return time series. Furthermore, the skewness value of -0.1491113 indicates a slight asymmetry in the left tail of the distribution. Concurrently, the kurtosis value registered at 8.190411, a figure that significantly exceeds the Gaussian reference value, thereby indicating a non-Gaussian distribution. This assertion is further reinforced by the Jarque-Bera normality test, whose p-value is notably lower than the critical threshold of 5%.

Table 1 Descriptive statistics of MASI logarithmic returns

Mean	Sd	Skewness	Kurtosis	Jarque-Bera	Normality Test
0.004026	0.2343612	-0.1491113	8.190411	Statistic	P-value
				3646.7	$2.2 \times 10^{-16}$

Prior to testing for the presence of long-memory dependencies, the MASI yield series was subjected to the simplified strategy of the Augmented Dickey-Fuller test.

The results presented in Table 2 demonstrate that for all three models, the ADF test statistics are consistently below the critical values at all three significance levels. Consequently, we can conclude that the MASI yield series under consideration is stationary, thereby rejecting the hypothesis of the presence of a unit root at the 1% threshold.

In this study, we aim to investigate the characteristics of long-memory dependencies in the series of returns of the main index of the Moroccan stock market. In order to further characterize the dependency structure of the studied series, we employ the ARFIMA modeling.



**Table 2** Results of the Augmented-Dickey-Fuller (ADF) test.

Models	ADF Test's statistic	Critical values		
		1%	5%	10%
M3	-34.7686	-3.96	-3.41	-3.12
M2	-34.7739	-3.43	-2.86	-2.57
M1	-34.7566	-2.58	-1.95	-1.62

The results of Table 3 indicate that the estimated parameters of the ARFIMA model (2, 0 .58675, 3) are all significant at the 10% level. However, the diagnostic tests on the residuals of the ARFIMA model estimated in our case are subject to both the problem of heteroscedasticity and non-normality. Consequently, it is not possible to determine whether the ARFIMA model is unsuitable for the data employed, or whether the market is inefficient. This is an instance of the problem of the joint hypothesis.

**Table 3** ARFIMA model estimation.

Parameter	<i>d</i>	AR (1)	AR (2)	MA (1)	MA (2)	MA (3)
Estimation	0.058675	0.509473	0.012321	0.244334	0.115303	0.076499
P-value	0.000917 ***	2x10 <sup>-16</sup> ***	0.099867 *	2x10 <sup>-16</sup> ***	2x10 <sup>-16</sup> ***	2x10 <sup>-16</sup> ***
Diagnostic Tests						
Test	Box-Ljung Test		ARCH LM Test		Jarque-Bera Test	
Statistics	2.3042		328.47		4321.3	
P-value	0.8056		2.2x10 <sup>-16</sup>		2.2x10 <sup>-16</sup>	

In order to remedy the expected problems, our methodology entails the co-estimation of the ARFIMA model with the FIGARCH model, using the following distributions; Normal, Student, Generalized Error and their skewed variants. Moreover, the Normal Inverse Gaussian, Generalized Hyperbolic, as well as Johnson's reparametrized SU distribution.

The information criterion and diagnostic tests on the residuals allowed us to select a single model with optimal specification for the data studied: the ARFIMA-FIGARCH model under the skew normal distribution.

First and foremost, the results obtained through the Nyblom stability test as presented in Table 4 demonstrate that all parameters of the ARFIMA (2, *d*, 3) FIGARCH (1, *D*, 1) model have values that lie below the critical value at the 1% threshold, indicating that the estimated model parameters are stable and demonstrate no structural change over time.

**Table 4** ARFIMA-FIGARCH model estimation.

Parameter	Estimation		P-value		Nyblom stability test	
	Akaike	Bayes	Shibata	Hannan-Quinn	Statistic	Critical value is 0.75 at the 1% threshold
$\mu$	0.000079		0.691457		0.45973	
<i>d</i>	0.079697		0.021625		0.72435	
AR (1)	-0.021803		0.974323		0.59400	
AR (2)	-0.267974		0.579635		0.05131	
MA (1)	0.188015		0.789736		0.57934	
MA (2)	0.284127		0.640520		0.06004	
MA (3)	-0.011185		0.927432		0.06290	
$\omega$	0.000006		0.000363		0.69912	
$\alpha$	0.000000		1.000000		0.05405	
$\beta$	0.317401		0.000000		0.20850	
<i>D</i>	0.659347		0.000000		0.49679	
Skew	1.028846		0.000000		0.41555	
Information criterion	Akaike	Bayes	Shibata	Hannan-Quinn	Critical value is 0.75 at the 1% threshold	
	-7.1665	-7.1440	-7.1665	-7.1584		

As indicated in Table 5, several results of significant importance were revealed with regard to the diagnostic tests performed on the residuals of the estimated ARFIMA-FIGARCH model. Weighted Ljung-Box tests on standardized residuals and squared standardized residuals indicate that there is no autocorrelation at all lag levels at the 1% threshold. Furthermore, the results of the weighted ARCH LM test indicate that the hypothesis of homoscedasticity of residuals from lag 3 to lag 7 at the critical 1% threshold is accepted. Furthermore, the results of the sign bias test indicate that the residuals of the estimated model do not exhibit any sign bias, nor any negative or positive signs, at the 1% threshold. The impact of global sign biases (joint) is also statistically insignificant, indicating that the simultaneous appearance of positive and negative sign biases has no substantial effect on the residuals of the estimated ARFIMA FIGARCH model. These findings demonstrate that the estimated model is capable of accurately representing the underlying structure of the data, indicating a robust model specification.

Once the robustness of the estimated model has been verified, it is of great significance to shed light on the long-memory parameters. By focusing on both the fractional differentiation parameters in the estimated ARFIMA-FIGARCH model,



it becomes possible to analyze the existence of long-memory dependencies in both the returns and volatility of the main Moroccan stock market index. The results presented in Table 4 indicate that the parameter  $d$  is of the order of 0.079697 and is statistically significant at the 5% level. This result corroborates the existence of long-term dependencies in yields. In our case, this implies that past values exert some influence on future returns, and the value of parameter  $d$  indicates that this influence diminishes rapidly over time. Concurrently, the value of parameter  $D$  is estimated at 0.659347, which is statistically significant at the 1% level. This indicates the presence of a robust long-term dependence in the volatility of returns on the Moroccan All Shares Index.

**Table 5** Diagnostic tests of the ARFIMA-FIGARCH model

Weighted Ljung-Box Test on Standardized Residuals		
Lag	Statistic	P-value
Lag [1]	0.7524	0.3857
Lag [14]	7.8169	0.2913
Lag [24]	12.2995	0.5021
Weighted Ljung-Box Test on Standardized Squared Residuals		
Lag	Statistic	P-value
Lag [1]	0.3853	0.5348
Lag [5]	2.0319	0.6109
Lag [19]	3.1209	0.7388
Weighted ARCH LM Tests		
Lag	Statistic	P-value
Lag [3]	1.354	0.2446
Lag [5]	2.438	0.3823
Lag [7]	2.989	0.5156
Sign Bias Test		
	T-value	P-value
Sign Bias	1.3895	0.1648
Negative Sign Bias	0.8159	0.4146
Positive Sign Bias	0.7931	0.4278
Joint Effect	4.7465	0.1913

Let's assume, absurdly, that the Moroccan stock market is efficient. This implies that prices on this market incorporate all available information, including historical prices, instantaneously. This is tantamount to assuming the absence of long-term dependencies, both in terms of returns and the volatility of returns on this stock market. However, and in view of the results that confirm the robust specification of the ARFIMA-FIGARCH model chosen for the mobilized data. We can now turn to the analysis of the long run dependency parameters. In our case, these parameters clearly show the presence of long-term dependencies on both the returns and the volatility of the returns of the main Moroccan stock market index.

Therefore, beyond the joint hypothesis problem, ARFIMA-FIGARCH modeling has provided insights that call into question the efficiency of the Moroccan stock market. Using this approach, our results suggest that past returns and past volatility variations have a persistent influence on future returns and volatility levels, respectively, taking into account both the long-term persistence of returns (the ARFIMA component) and conditional volatility (the FIGARCH component).

Our estimation results reveal a moderate persistence of shocks on the returns of the main Moroccan index "MASI", accompanied by stronger long-memory dependencies on the volatility of the Moroccan stock market. The findings of the present study stand in contrast to those of Anoruo et Gil-Alana (2010), while aligning with and extending, in the Moroccan context, the conclusions regarding long-term memory highlighted by Omerani et al. (2022). This configuration follows perfectly the estimation results of the ARFIMA (1,  $d$ , 1) - FIGARCH (1,  $D$ , 1) model in the Chinese context (Tripathy, 2022). Our results also support the existence of a double memory (at the level of returns and the volatility of stock market returns), which has been highlighted by the existing financial literature, specifically the work of Boubaker et al. (2022), Kasman et al. (2009), Nguyen et al. (2020) and Nkrumah Ababio et al. (2019). On the other hand, there are some specificities in the Moroccan stock market context: the ARFIMA-FIGARCH model has a robust specification under the asymmetric normal distribution. This result contradicts other studies, notably Basira et al. (2024), who argue that other probability distributions (t-Student, t-skewed and GED) are appropriate in their specific contexts. These results highlight the importance of evaluating various probability distributions when estimating ARFIMA-FIGARCH models. This is with a view to selecting the optimal distribution to model the dual-memory characteristics of financial time series, the normality assumption of which is often rejected. This approach, which takes account of these different data specifications, clearly allows us to call into question the market efficiency hypothesis, while overcoming the associated limitations of the joint hypothesis problem.

Finally, our findings call into question the hypothesis that the Moroccan stock market is efficient, opening up potential opportunities for market participants. This inefficiency implies the possibility of generating abnormal performance by exploiting the predictability of price movements. However, even arbitrage to correct these inefficiencies can be very limited, as long memory makes this process difficult and risky. Investors who can correctly identify and model this long memory can



gain an informational advantage over the market, enabling them to more accurately anticipate future price trends and achieve better performance. In this context, active portfolio management can become more relevant, as active managers with appropriate models can outperform passive strategies. While effectively exploiting long memory remains a complex challenge requiring advanced modeling and risk management skills, it offers interesting opportunities for sophisticated investors.

#### 4. Conclusions

Academic researchers, regulators, and practitioners have consistently demonstrated a keen interest in the efficiency of stock markets. The question of market efficiency, which has remained a persistent concern, has garnered even greater attention in the current era, particularly during periods of market distress. This heightened interest can be attributed to the lack of definitive conclusions and the inherent challenges associated with the joint hypothesis problem of market efficiency. The difficulty in determining whether the model employed is inadequate or the market itself is inefficient further contributes to the ongoing interest. In this context, we have overcome the limitations of the standard ARFIMA model, which is subject to the problem of non-normality and heteroscedasticity, by resorting to ARFIMA-FIGARCH estimation with a rigorous model specification.

Our analysis focuses on the Moroccan stock market, represented by its main MASI index, over the period from January 2, 2013 to December 29, 2023. The results of our study, particularly the ARFIMA-FIGARCH model estimated under skew normal distribution, demonstrated the existence of moderate long memory in the returns, along with significant persistence in the volatility of the Moroccan stock market. These results indicate the presence of long memory dependencies in both returns and return volatility, which calls into question the efficiency of the Moroccan market, beyond the joint hypothesis problem.

In practical terms, this persistence, whether in terms of returns or volatility, can be exploited by informed participants who are able to implement forecasting strategies based on long-memory models. Consequently, these investors could profit from the delayed adjustment of stock prices, generating excess returns based solely on historical price movements. However, this form of predictability can also serve as a catalyst for information asymmetries, which have the potential to challenge the fairness of the market, particularly for small investors or less sophisticated market participants. In this respect, ARFIMA-FIGARCH models can be integrated into the decision-making process to improve risk management by anticipating prolonged periods of volatility, thus providing an effective opportunity to exploit persistent trends.

At the macroeconomic level, the persistence of shocks, particularly during periods of systemic risk, can pose a challenge to the financial system's capacity to adapt to new economic circumstances. However, in a context where the economy requires an efficient shock adjustment mechanism to avert the exacerbation of the situation, this slow response contributes to the system's fragility. Consequently, enhancing transparency, regulation, and financial education emerges as a pivotal factor in optimizing its efficiency and resilience. In this sense, authorities could better understand market dynamics by adopting models that incorporate ARFIMA-FIGARCH. This would facilitate targeted interventions to mitigate systemic risks.

Finally, this research contributes to a more comprehensive comprehension of stock markets by considering long-term dependencies in returns and persistent shocks in volatility. Investors will be better positioned to make informed decisions with a deeper understanding of how effectively the Moroccan stock market performs. They will be able to capitalize on potential opportunities to outperform the market while rigorously mitigating the inherent risks associated with a complex and constantly evolving financial landscape.

#### Ethical considerations

This study did not involve human participants, clinical data, or personal information. The analysis is based exclusively on publicly available financial data of the MASI index, sourced from Investing.com. Therefore, ethical approval and informed consent were not required. The study complies with the Multidisciplinary Science Journal's policies on the ethical use of data.

#### Conflict of Interest

The authors declare no conflicts of interest.

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