


# Analysis of learning obstacles in grade 1 elementary mathematics using the concrete-pictorial-abstract (CPA) approach



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**Abstract** This study aims to identify and analyze the learning obstacles faced by first-grade elementary school students in mathematics learning using CPA (Concrete-Pictorial-Abstract) approach-based textbooks. The CPA approach is considered effective in helping students understand mathematical concepts gradually, starting from the concrete stage, moving to the pictorial stage, and finally reaching the abstract stage. However, in its implementation, various challenges and difficulties may still be encountered by both students and teachers. This research will be conducted randomly in several Grade 1 classes at Sekolah Lentera Harapan, which are confirmed to be using CPA-approach, namely the Matematika Juara book that adopts the Singapore curriculum. Through qualitative methods and direct observation, this study seeks to gain a deep understanding of the types of learning obstacles experienced by students and to explore the factors that influence the effectiveness of CPA-approach in teaching and learning activities. The expected outcomes of this research include several key aspects. First, documentation of the types of learning obstacles found among students when using CPA-based textbooks. Second, the formulation of more effective teaching strategies to overcome the identified learning obstacles, thereby improving students' understanding of mathematics. Third, a practical guide for teachers on how to optimally implement CPA-based books in Grade 1 classrooms. It is hoped that this study will make a significant contribution to the development of curriculum and mathematics teaching methods, particularly in Grade 1, by promoting more optimal use of the CPA approach.

**Keywords:** learning obstacles, grade 1 elementary mathematics, concrete-pictorial-Abstract (CPA) approach, analysis of learning barriers

## 1. Introduction

Mathematics is one of the core subjects that plays a vital role in developing students' logical and analytical thinking skills from an early age. At the elementary school level, the understanding of fundamental mathematical concepts serves as a foundation for critical thinking and problem-solving abilities in subsequent stages of education (NCTM, 2000). However, despite its importance, mathematics instruction at the elementary level often faces various challenges. First-grade students in particular frequently encounter difficulties in understanding mathematical content, which can negatively impact their learning capacity and overall development. One factor that may influence students' understanding of mathematics is the presence of learning obstacles encountered during the instructional process (Brousseau, 1997).

The use of the Concrete-Pictorial-Abstract (CPA) approach in mathematics education is believed to help address such learning obstacles. Jerome Bruner first introduced the CPA approach as part of his theory of representation, which states that children learn through three stages of representation: enactive (concrete), iconic (visual), and symbolic (abstract) (Bruner, 1996). The CPA method is designed to support students' understanding of mathematical concepts progressively from manipulating concrete objects, to representing them visually, and ultimately transitioning to abstract conceptualization. This method aims to foster deep conceptual understanding through a developmentally appropriate instructional sequence (Piaget, 1952). CPA-based textbooks offer strategies that emphasize these stages, allowing students to internalize mathematical concepts in a step-by-step manner (Moyer, 2001). However, although the CPA approach holds significant promise, its implementation in classrooms particularly in Grade 1 often encounters barriers that require further investigation.

In elementary schools, especially in first grade, students tend to show significant differences in comprehension levels and learning pace, which frequently pose challenges to the effective application of the CPA approach. Learning obstacles



experienced by students may arise from various factors, ranging from cognitive challenges, such as difficulties in grasping abstract representations (Tall, 2004), to psychomotor limitations, such as difficulty converting concrete information into visual representations. Additional obstacles may also stem from affective factors, including lack of confidence or anxiety when faced with mathematical problems (Ashcraft & Krause, 2007).

Based on the aforementioned issues, there is a need for an in-depth investigation into the learning obstacles faced by Grade 1 students in mathematics instruction using CPA-approach. This study focuses on analyzing the types of learning obstacles encountered by students and examining how the use of CPA-approach influences the teaching and learning process. Sekolah Lentera Harapan, which has adopted the CPA Juara textbook as part of its formal instructional approach in Grade 1 mathematics, was selected as the research site. Utilizing a qualitative research method that includes direct observation and interviews, this study aims to provide insights into the effectiveness of CPA-based materials in addressing learning obstacles, as well as the factors that influence the successful implementation of this approach in the classroom.

## 2. Materials and Methods

### 2.1. Qualitative Research Method

A qualitative approach is highly appropriate for this study because it allows for a rich, contextualized understanding of the learning obstacles encountered by students in mathematics learning through the CPA (Concrete–Pictorial–Abstract) approach. This method emphasizes the exploration of meaning and experiences rather than numerical measurement, making it particularly suitable for capturing the complexity of classroom interactions, students' cognitive processes, and teachers' instructional practices (Babchuk, 2017; Creswell, 2017). By employing a qualitative design, the researcher can deeply investigate the perceptions, beliefs, and lived experiences of both students and teachers, uncovering nuanced insights that might be overlooked by quantitative methods (Patton, 2015). Moreover, qualitative inquiry facilitates the identification of specific pedagogical, cognitive, and affective factors that either enhance or hinder the effectiveness of CPA-based textbooks in mathematics instruction. This approach also allows for the integration of triangulation techniques, such as interviews, classroom observations, and document analysis, to strengthen the credibility and trustworthiness of the findings (Lincoln & Guba, 1985).

### 2.2. Case Study Design

A case study design can be applied by selecting several classes or schools as samples for the research. In this case, the focus of the study is on Grade 1 at Sekolah Lentera Harapan, which has implemented CPA-based textbooks. The case study provides a detailed picture of the learning context, students' interactions with the material, and classroom dynamics that may influence learning obstacles (Yin, 2018).

### 2.3. Data Collection Techniques

**Classroom Observation:** Direct classroom observation provides valuable insights into student behavior when using CPA-based textbooks, their responses to the material, and the difficulties they encounter at each stage of CPA (concrete, pictorial, and abstract). The observation also records teacher-student interactions in overcoming learning obstacles (Merriam & Tisdell, 2016).

**In-depth Interviews:** Interviews are conducted with both teachers and students to gain deeper insights into their experiences using CPA-based textbooks. Interviews allow the researcher to understand teachers' strategies in addressing learning obstacles and gather students' perspectives on their experiences (Patton, 2015).

**Documentation:** The collection of documents such as lesson plans, student work, or teachers' daily notes provides additional data to identify patterns of learning obstacles and students' responses to the CPA approach (Bowen, 2009).

Data collection was conducted over a period of three months (March–May 2025) to allow for multiple cycles of observation, interviews, and document analysis. The timeline was as follows:

**March 2025:** Initial coordination with the school, obtaining informed consent from participants, and conducting preliminary classroom observations to understand the teaching context.

**April 2025:** Classroom observations of mathematics lessons implementing the CPA approach (four sessions, one per week). Collection of students' written work during CPA-based mathematics lessons for further analysis of problem-solving strategies and misconceptions. First round of semi-structured interviews with the mathematics teacher focusing on instructional practices and perceived learning obstacles.

**May 2025:** Follow-up classroom observations to confirm patterns identified in earlier sessions. Second round of teacher interviews for clarification and elaboration of emerging themes. Brief follow-up interviews with selected students to validate interpretations from observations and written work analysis.

During the second round of interviews, the responses from the teacher and students began to repeat previously identified themes such as difficulties in transitioning from the pictorial to abstract stage, language barriers in word problems,

and differences in conceptual understanding among achievement levels. This repetition indicated that further interviews were unlikely to produce additional insights, thus confirming saturation.

#### 2.4. Data Analysis

**Thematic Analysis:** This analysis is used to identify key themes in the data, such as types of cognitive, affective, or psychomotor obstacles experienced by students, as well as factors affecting the success of using CPA (Braun & Clarke, 2006).

**Data Triangulation:** Triangulation is carried out by comparing data from observations, interviews, and documentation to ensure the validity and accuracy of the research findings (Denzin, 1978). This approach strengthens the reliability of the analysis and provides a more comprehensive picture. A qualitative approach is highly appropriate for this study because it allows for a rich and contextualized understanding of the learning obstacles encountered by students in mathematics learning through the CPA (Concrete–Pictorial–Abstract) approach. This study employs a qualitative design by analyzing three main sources of data: (1) students' written work, which reveals their problem-solving strategies, conceptual understanding, and common misconceptions; (2) classroom observations, which provide insights into the learning process, teacher–student interactions, and the implementation of CPA-based instructional strategies; and (3) in-depth interviews with teachers, which capture professional reflections, pedagogical considerations, and perceptions of students' learning challenges. By integrating these data sources, the researcher can triangulate findings to increase the credibility and trustworthiness of the research (Lincoln & Guba, 1985). The analysis of students' work enables the identification of specific misconceptions or gaps in understanding (Cai et al., 2020), while classroom observations offer rich contextual information about instructional delivery and student engagement (Creswell & Creswell, 2017). Teacher interviews, in turn, provide an insider perspective on pedagogical decision-making and the challenges faced in implementing CPA-based mathematics teaching (Patton, 2015). Together, these qualitative techniques allow for a comprehensive and in-depth exploration of the factors influencing the success or challenges of using CPA-based textbooks in mathematics instruction.

#### 2.5. Development of Recommendations

After analyzing the data, the researcher formulates practical recommendations for teachers to address learning obstacles in CPA-based mathematics instruction. These recommendations are developed based on empirical data and are expected to be relevant for application in primary education contexts (Miles et al., 2014).

#### 2.6. Sampling Techniques

This study employed purposive sampling to select participants who could provide rich and relevant information regarding the implementation of the CPA (Concrete–Pictorial–Abstract) approach in mathematics learning. Purposive sampling is particularly suitable in qualitative research because it allows the researcher to intentionally select individuals who have specific characteristics and experiences related to the research focus (Creswell & Creswell, 2017; Patton, 2015). The participants consisted of students and mathematics teachers from one elementary school that had adopted CPA-based mathematics instruction. Student selection criteria are actively enrolled in the first grade level where CPA-based mathematics is implemented. Willingness to participate and provide informed consent from both students and from the school. Teacher selection criteria are mathematics teachers who have been implementing the CPA approach in their teaching for at least one academic semester, willing to participate in interviews, classroom observations, and allow the analysis of students' work beside that possessing experience in using CPA-based textbooks in daily instruction. The final sample comprised two mathematics teacher and three students (three high-achieving, three medium-achieving, and three low-achieving). This distribution was intended to ensure representation of various learning profiles and to enable the identification of learning obstacles across different achievement levels.

### 3. Results and Discussion

This study on the analysis of mathematics learning obstacles among Grade 1 students was conducted at a private elementary school located in Labuan Bajo, Flores, East Nusa Tenggara (NTT), Indonesia. The school was selected randomly, considering the accessibility for the researcher and the willingness of the institution to collaborate in the study. Additionally, the compatibility of schedules between the researcher and the educational institution was also a determining factor. The selected school has a unique background, having experienced rapid growth in recent years and becoming one of the most sought-after schools in the Labuan Bajo area. Its strategic location in the heart of one of NTT's most prominent tourist destinations significantly influences the dynamics of classroom learning. Labuan Bajo is widely recognized as the gateway to Komodo National Park, a UNESCO World Heritage Site (see in Figure 1). The area not only offers breathtaking natural attractions such as Komodo Island, Pink Beach, and Wae Rebo Village, but also serves as a hub for economic, cultural, and educational activities in the region. In light of the rapid development of tourism, schools in this area face both challenges and opportunities in creating an optimal learning environment for students, including in the context of mathematics education. The evolving

social and economic landscape of Labuan Bajo necessitates adaptive teaching approaches to ensure that foundational subjects such as mathematics remain relevant, engaging, and meaningful for young learners.



**Figure 1** Iconic Photos of Labuan Bajo Indonesia.  
 Source: <https://explore.airasia.com/>.

### 3.1. Interview Findings with Two Grade 1 Elementary School Teachers

Table 1 presents the interview findings with two Grade 1 elementary school teachers regarding the epistemological obstacles faced by students in understanding mathematics.

**Table 1** Interview Findings with Two Grade 1 Elementary School Teachers.

Indicator	Question	Teacher 1 Response	Teacher 2 Response
Epistemological Obstacles	In your opinion, which mathematical concepts are most difficult for first-grade students to understand? Why?	The concept of time, because it requires complex understanding to grasp the meaning of minutes, seconds, hours, and the function of the hour and minute hands on a clock.	Arithmetic operations (addition and subtraction) are among the most difficult concepts. Students often struggle to distinguish between addition and subtraction, especially when problems are abstract.
	Do students often experience conceptual errors (e.g., assuming a larger number always means “more”)? Can you give an example?	Yes, some misunderstandings still occur, such as distinguishing “more” and “less” in word problems.	A common conceptual error is that students often solve subtraction problems as if they were addition. They also often confuse the function of the hour and minute hands on a clock.
	How do students typically respond when first introduced to addition and subtraction?	They tend to understand addition fairly well at first because they are used to counting forward. Subtraction is more difficult as they are less familiar with counting backward. They also get confused with the borrowing method in vertical subtraction.	Most students are excited because they already have some understanding of addition and subtraction, but some students are still confused about the concepts.
	What strategies do you use to correct students’ misconceptions about basic math concepts?	I use manipulatives or contextualized word problems that relate to students’ daily lives.	Strategies I use: 1) Analyze students’ misconceptions, 2) Reinforce simpler, foundational concepts, 3) Apply the CPA approach.
	What is your students’ background? Do they attend tutoring? Do most of them learn math quickly?	50% of my students attend tutoring. Most of them understand math quickly.	None of my students attend tutoring. About 61.9% (13 students) are quick to understand math.
Ontological Obstacles	What types of problems are most confusing for your students? Can you give an example?	Word problems. Students have difficulty translating the problem into mathematical form.	Word problems involving operations or time calculation.
	Do students find it hard to understand abstract math concepts (such as numbers or operations)? If so, how do you help them understand?	Students are often confused with subtraction. I try various methods such as number bonds.	Students find abstract math concepts difficult, especially in operations. I help them by: - Using physical manipulatives, - Playing simple math games, - Using number lines, - Repeating basic concepts.



	<p>Do you use concrete media (e.g., beads, pictures, physical objects) in your teaching? How effective do you find them?</p> <p>How do students react when math is taught without real-life examples? Do they tend to be confused or frustrated?</p>	<p>Yes. For operations, I use small origami paper in two colors. For geometry, I use simple 3D nets. For time, I use a clock model with movable hands.</p> <p>Yes, they tend to be confused.</p>	<p>Yes, I often use concrete media. I find them very effective for teaching math.</p> <p>They tend to be confused.</p>
Didactical Obstacles	<p>What teaching methods do you use most frequently for first-grade math?</p> <p>Have you tried problem-based learning? If yes, what was the main challenge?</p> <p>Do you think the available teaching materials (textbooks, worksheets, digital content) meet the needs of first-grade students? If not, what is missing?</p> <p>Do you feel you have enough time to prepare interactive and engaging math lessons?</p> <p>What challenges do you face in designing enjoyable math lessons for Grade 1 students?</p> <p>Have you ever used local NTT cultural elements to teach first-grade math? If yes, what objects or visuals did you use? What support do you need to overcome challenges in teaching first-grade math? (e.g., teacher training, manipulatives, clearer curriculum)</p> <p>Have you ever implemented a specific strategy or innovation that successfully improved students' math understanding? If so, please describe.</p>	<p>Project-Based Learning and Inquiry.</p> <p>Yes. The main challenge is classroom management. First-grade students still need a lot of detailed guidance and often ask questions before taking action. I think the materials are appropriate, but it would be better if they included simple activities students can do directly, such as arranging patterns or assembling 3D shapes from nets.</p> <p>If using regular work hours, time is insufficient due to the school's busy agenda and event preparation. I usually work overtime to make time.</p> <p>My class is diverse. Some students learn quickly (often with extra tutoring), while others need slow, careful instruction. Fast learners may get bored if the material is repeated, but slower learners will fall behind if the pace is too fast.</p> <p>Never.</p> <p>Simple activity references for Grade 1, manipulatives, and teacher training.</p> <p>I invited students to do simple activities like creating patterns or assembling 3D shapes from nets (cube, cuboid, cone). They enjoyed the activities and were able to identify shapes and attributes directly.</p>	<p>Discussion method, lecture method, and conceptual learning.</p> <p>Yes, I have tried it. The main challenge is that students take a long time to analyze the problem and understand the context. I think some parts are lacking, especially in explaining pictograms. The printed book does not cover this material in enough detail.</p> <p>I believe the time provided is sufficient for preparing lessons.</p> <p>So far, I haven't faced major challenges. The only issue I've had is limited materials for creating teaching media, but I found alternative solutions.</p> <p>Yes. I used traditional NTT patterned fabric (Songke cloth) to introduce visual patterns (non-numerical).</p> <p>I need teacher training and manipulatives.</p> <p>My strategies and innovations have not been fully successful, but they have helped students gradually improve their understanding of math.</p>

### 3.2. Examples of Student Work

The task in Question 1 (see in Figure 2) required the student to observe and complete visual patterns by placing a cross (X) on the correct missing image in each sub-question (1a–1c). The student's performance reveals various types of learning obstacles:

#### 3.2.1. Didactical Obstacles (related to instruction and task design)

Item 1.a: The student correctly identified the missing pattern element but did not follow the specific instruction to “mark with a cross (X).” Instead, they drew and circled the image, which reflects a misinterpretation of the task format. This suggests the student may not have fully understood the instructional language or was influenced by prior experiences where similar tasks used different marking conventions. Teacher feedback written on the page (“kurang memahami instruksi”, “did not understand the instructions well”) supports this interpretation. The instructional design (or its delivery) may not have



sufficiently supported students' understanding of expected task behavior, especially for early-grade learners still developing test-taking routines.

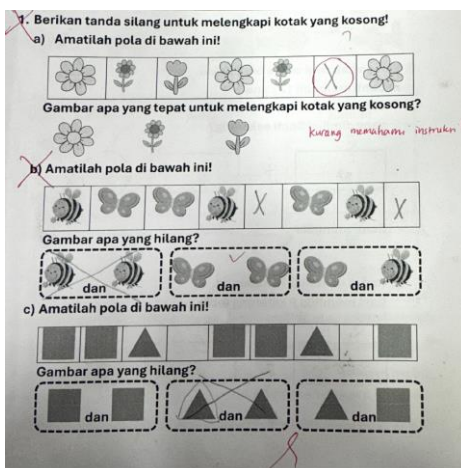


Figure 2 Student's Response to Question 1.

3.2.2. Epistemological Obstacles (related to students' understanding of concepts)

Item 1.b: The student incorrectly identified the missing image in the pattern. This error indicates a misconception about repeating visual sequences, possibly due to a surface-level understanding of the images rather than recognizing underlying structure or pattern rules. The choice may have been based on visual similarity or position rather than pattern logic, reflecting a limited conceptual grasp of sequencing and patterning, which are foundational in early mathematics learning.

3.2.3. Ontogenic Obstacles (related to cognitive development and readiness)

Item 1.c: The student correctly identified the missing shape but again used inconsistent marking, including circling and placing the answer outside the expected area. This may reflect developmental limitations in fine motor control, attention to multi-step instructions, or spatial awareness common in early primary students. Although the pattern logic was understood, the execution did not align with the format expected, suggesting a gap between conceptual understanding and performance execution.

Based on Figure 3, here is a detailed learning obstacle analysis of Student's responses to Question 2, using the epistemological, didactical, and ontogenic framework. Question 2 is about basic addition and subtraction (number operations 1–20). This section consists of basic number sentences involving addition and subtraction. The student made frequent calculation errors in this section:

- Item 2a: Correct.
- Items 2b, 2c, 2d, and 2e: Incorrect. The student answered:
  - 2b).  $8 + 12 = 22$  (incorrect).
  - 2c).  $18 - 6 = 17 \rightarrow$  wrong (correct answer: 12).
  - 2d).  $7 + 9 = 16 \rightarrow$  correct.
  - 2e).  $17 - 8 = 10 \rightarrow$  wrong (correct answer: 9).
  - 2f).  $20 - 6 = 14 \rightarrow$  wrong (correct answer: 6).

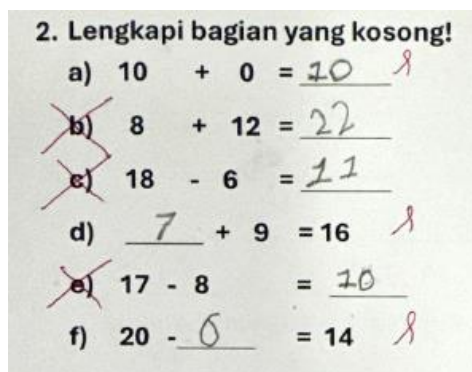


Figure 3 Student's Response to Question 2: Basic Addition and Subtraction (Number Operations 1–20).

### 3.2.3.1. Identified Obstacles

#### 3.2.3.1.1. Epistemological Obstacles

The student seems to rely on memorization or estimation rather than solid conceptual understanding. For example, the error in  $17 - 8 = 10$  indicates a misunderstanding or misapplication of subtraction facts.

#### 3.2.3.1.2. Ontogenic Obstacles

Multiple basic errors suggest the student may not yet have fluency in number operations within 20, a developmental stage common for early primary students. This includes difficulties with mental computation and number sense.

#### 3.2.3.1.3. Didactical Obstacles

The variety in how operations are presented (horizontal equation, missing element format) may contribute to confusion if students are not regularly exposed to these representations during instruction.

Question 3 Figure 4 is about word problems.

3a. "Santi bought 12 pencils. She already had 5 pencils. How many pencils does Santi have now?". The student incorrectly wrote 14, indicating the operation performed was  $12 - 5$  instead of  $12 + 5$ .

3b. "Andre has 18 marbles. Bani has 9 fewer marbles than Andre. How many marbles does Bani have?". Student answered correctly: 9 marbles, using  $18 - 9 = 9$ .

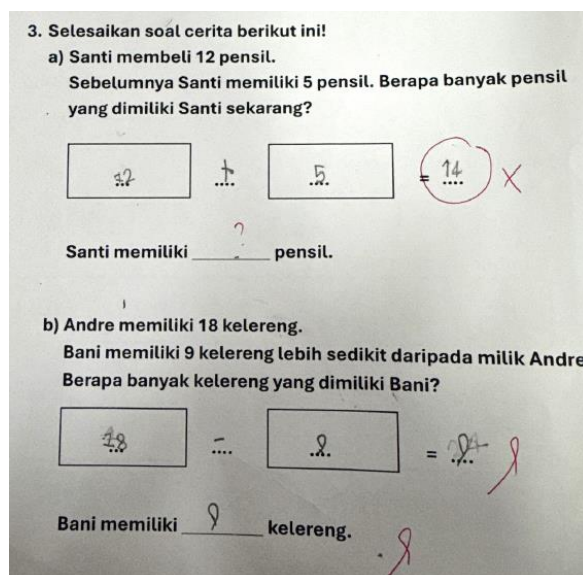


Figure 4 Student's Response to Question 3: Word Problems.

### 3.3. Identified Obstacles

#### 3.3.1. Epistemological Obstacles

In 3a, the student misunderstood the structure of the story problem. Rather than recognizing that the total number of pencils should be the sum of existing and newly bought pencils, the student subtracted. This reflects a relational misunderstanding of story context and the required operation.

#### 3.3.2. Didactical Obstacles

The presentation format may not be fully aligned with students' everyday experiences or the examples they have encountered in class. If problems are not consistently modeled in class, students may revert to default strategies (like always subtracting when two numbers are present).

#### 3.3.3. Ontogenic Obstacles

In 3a, the student's choice may also reflect overgeneralization (i.e., assuming subtraction is always the operation in "how many now" contexts). This kind of reasoning is common in early stages of cognitive development, where students apply simple rules without fully interpreting context.

Student demonstrates a combination of conceptual misunderstanding, misinterpretation of problem structure, and inconsistent calculation skills, suggesting the presence of epistemological obstacles (misunderstanding mathematical operations and their meanings), didactical obstacles (possible misalignment in teaching and assessment design), and ontogenic obstacles (developmental readiness in abstract reasoning and symbolic manipulation).

Based on Figure 5, here is an academic-style analysis of Student’s response to Question 4, focusing on learning obstacles using the ontogenic, epistemological, and didactical framework. Students were asked to create tally marks (turus) from a pictogram, to count and record quantities and compare quantities using comparative phrases: more than, less than, or the same as. This is the student’s performance summary.

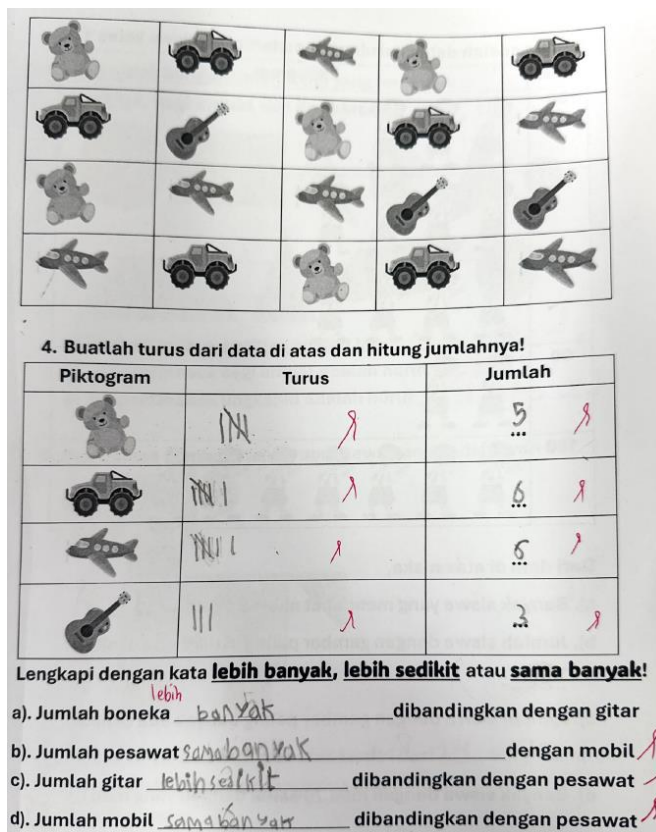


Figure 5 Student’s Response to Question 4: Data Grouping and Comparison.

Part 1: Tally Marks and Counting

Tally Representation.

Teddy bears: Correct (5 tallies).

Cars: Correct (6 tallies).

Airplanes: Correct (6 tallies).

Guitars: Correct (3 tallies).

Total Counts: All accurately written to match the pictograms.

Student demonstrated accurate representation of visual data into symbolic tally form and correctly performed simple one-to-one correspondence.

Part 2: Comparative Statements.

a. More teddy bears than guitars → Correct ("lebih banyak").

b. Same number of airplanes and cars → Correct ("sama banyak").

c. Fewer guitars than airplanes → Correct ("lebih sedikit").

d. Same number of cars and airplanes → Correct ("sama banyak").

From this, it can be seen that student demonstrated strong procedural understanding in data grouping and tally construction. Beside that partial conceptual understanding in data comparison, limited by relational language interpretation.

Based on Figure 6, here is a structured academic style description and analysis of Student’s response to Question 5, viewed through the lens of learning obstacles. In this part, students were asked to interpret data from a pictogram representing the number of Grade 1 students who scored different marks in a math test (60, 70, 80, 90, 100), and to answer four data based questions. The student was asked to interpret pictogram data showing the distribution of math test scores among Grade 1 students and to answer questions related to data quantity and comparison (see in Table 2).

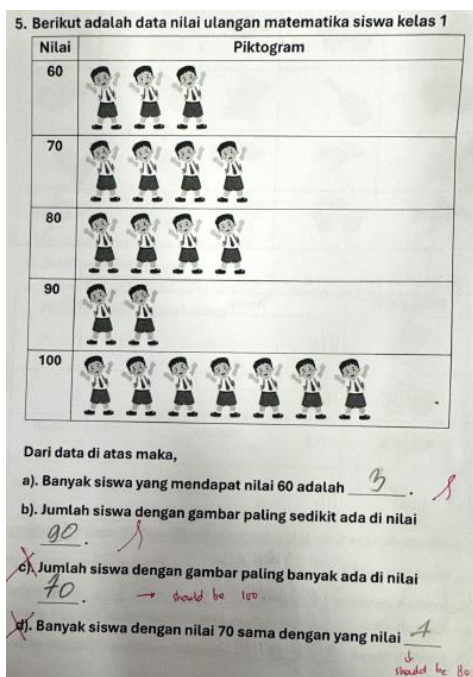


Figure 6 Student’s Response to Question 5: Interpreting Pictogram Data.

Table 2 Student’s Responses in Question Number 5.

Question	Expected Answer	Student’s Answer	Correct?	Notes
a) Number of students with a score of 60	3	3	Correct	Accurate comparison
b) Fewest number of students is at score...	90 (2 students)	90	Correct	Accurate comparison
c) Most students are at score...	100 (7 students)	70	Incorrect	Misinterpretation of quantity
d) Number of students scoring 70 is equal to...	80 (both = 3)	7	Incorrect	Incorrect comparison

From student’s responses, it can be identified learning obstacles that

3.4. Epistemological Obstacles

The student demonstrates a misconception in interpreting pictorial data, such as in item (c), where 70 was incorrectly identified as having the highest frequency. In item (d), the student did not correctly recognize equal quantities (3 figures in both 70 and 80), which indicates difficulty in making accurate quantity comparisons. These errors show that the student has not fully developed the conceptual understanding of how pictograms represent data quantities, and may be relying on visual approximation rather than systematic counting.

3.5. Ontogenic Obstacles

The miscounting and misinterpretation may stem from limited visual attention, short working memory span, or underdeveloped comparative reasoning, all of which are typical in early learners. The student may also struggle with holding multiple representations in mind simultaneously, such as comparing the figures in 70 and 80 while referencing a different row like 60.

3.6. Didactical Obstacles

It is possible that instructional emphasis on pictograms and how to read them accurately has not been sufficiently reinforced. If students are not frequently exposed to pictogram tasks that require close observation and comparison, they may fail to develop the habit of careful counting or analytical comparison.

Student exhibited partial understanding of data comparison (as seen in item a and b), but significant difficulty in accurate data reading and interpretation (items c, d). Suggesting epistemological obstacles (conceptual misunderstanding of visual data), Ontogenic challenges (in attention and comparative reasoning), and possible didactical gaps in scaffolding or modeling of pictogram interpretation.

The task required students to identify letters representing Triangles, Quadrilaterals, and Circles (see Figure 7). Student’s answers a) Triangles: c, d, e → Correct, b) Quadrilaterals: b, g, h, i → Correct and c) Circles: a, f, j → Correct. The student demonstrated a clear and accurate understanding of basic geometric shapes, their properties, and classification. The answers



are all correct and show proficiency in visual shape recognition, categorization by geometric attributes, and counting composite figures. These responses reflect appropriate conceptual understanding expected at the primary level.

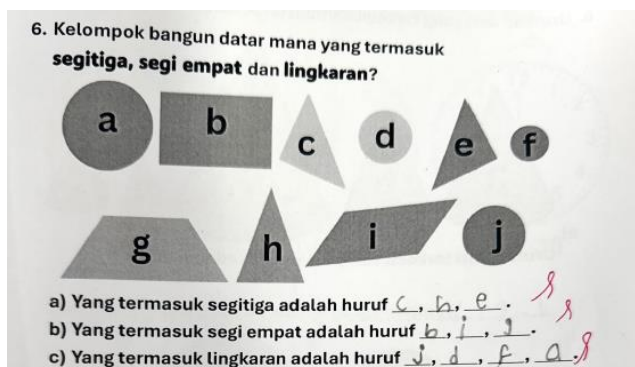


Figure 7 Student's Response to Question 6: Identifying Geometric Shapes.

The student was asked to observe a figure resembling a dog composed of multiple shapes and answer three related questions (see Figure 8). Student's answers are a) Total number of shapes: 14 → Correct, b) Number of curved shapes (circles): 2 → Correct, c) Number of triangles: 1 → Correct, and d) Number of quadrilaterals: 8 → Correct. The figure of the dog consists of 14 two-dimensional shapes, including multiple rectangles, two circles (for eyes and tail), one triangle (for the tail), and various squares/rectangles for the body, head, ears, and legs. The student accurately counted 2 circles (rounded shapes), 1 triangle (tail), 8 quadrilaterals (all other rectangular or square parts), and reached the correct total count.

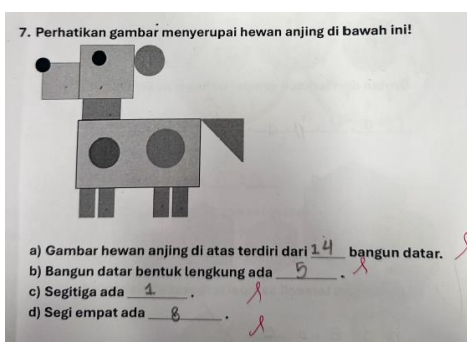


Figure 8 Student's Response to Question 7: Counting Shapes in a Composite Figure (Dog Image).

Based on Figure 9, here is an academic-style analysis of the Student's responses to Question 8, viewed through the lens of learning obstacles. The student was asked to observe three sets of three-dimensional objects (cones, spheres, and cubes) and arrange them in order from smallest to largest, based on their relative size as depicted.

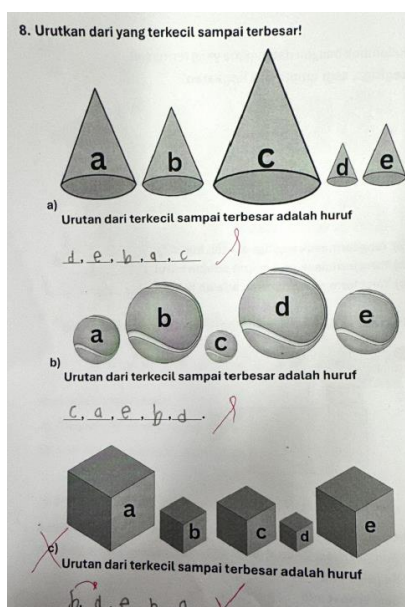


Figure 9 Student's Response to Question 8: Ordering 3D Shapes from Smallest to Largest.

From Table 3, it can be concluded that learning obstacle analysis from:

**Table 3** Students' Responses to Question Number 8.

Item	Correct Sequence	Student's Answer	Correct?	Notes
a) Cones	d, e, b, a, c	d, e, b, a, c	Correct	Fully accurate
b) Spheres	c, a, e, b, d	c, a, e, b, d	Correct	Fully accurate
c) Cubes	d, b, c, e, a	b, d, e, a, c	Incorrect	Several reversals in size ordering

**3.7. Epistemological Obstacles**

The student successfully applied reasoning to compare cones and spheres, demonstrating a solid understanding of size comparison in visual/spatial tasks. However, in item c (cubes), the student incorrectly sequenced multiple items, suggesting a misconception in spatial estimation or difficulty perceiving subtle differences in volume, especially when objects have similar shapes and orientations. This indicates a partial epistemological obstacle; the student has the concept of ordering by size but struggles with consistency in its application, especially with cubic forms.

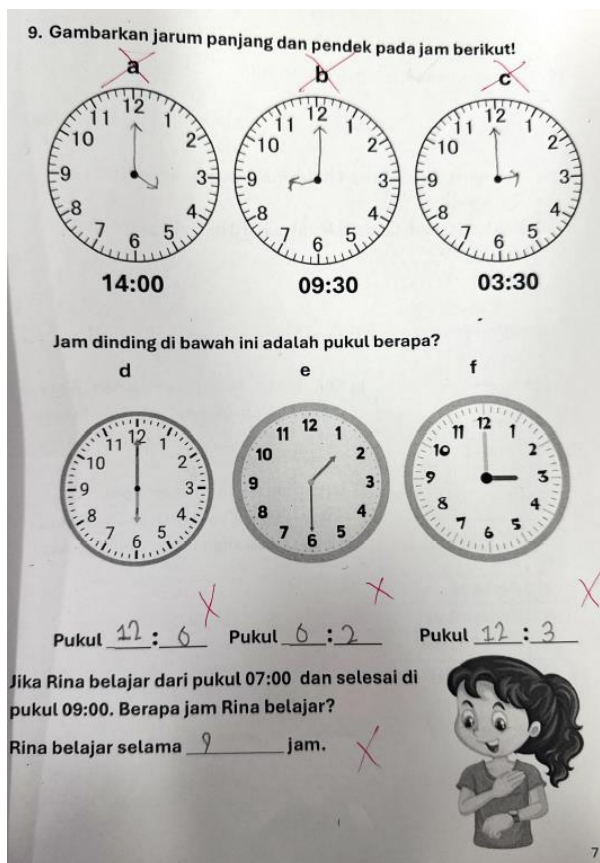
**3.8. Ontogenic Obstacles**

The errors in item c may also be influenced by developmental limitations in spatial perception and visual discrimination, which are still emerging at the early primary level. The cubes differ in subtle ways that may require more refined visual comparison skills. Younger learners may focus on height or position on the page, rather than actual volume.

**3.9. Didactical Obstacles**

If students have not had sufficient experience with manipulating 3D objects in real life, or if instruction has emphasized 2D representations only, they may not fully grasp volume or spatial differences just from images. The task design assumes a mature visual-spatial skill set that may still be developing in some students.

Based on Figure 10, here is an academic-style analysis of the Student's responses to Question 9, viewed through the lens of learning obstacles (epistemological, ontogenic, and didactical). This task assessed students' ability to draw clock hands to represent given times, read and write clock times from illustrations, and calculate elapsed time based on a simple word problem.



**Figure 10** Student's Response to Question 9: Understanding Time and Analog Clocks.

From Table 4, it can be identified learning obstacles that.

**Table 4** Students' Responses and Accuracy for Question 9.

Subtask	Expected Answer	Student's Answer	Correct?
a) Draw 14:00	Long hand at 12, short at 2	Incorrect	Hands are not differentiated
b) Draw 09:30	Long hand at 6, short between 9–10	Incorrect	Hands incorrectly placed
c) Draw 03:30	Long hand at 6, short between 3–4	Incorrect	Hands incorrectly placed
d) Read the clock	06:00	12:6	Incorrect notation
e) Read the clock	06:00	6:2	Incorrect notation
f) Read the clock	03:00	12:3	Incorrect
g) Duration calculation	2 hours (from 07:00 to 09:00)	9	Incorrect

**3.10. Epistemological Obstacles**

The student demonstrates a fundamental misunderstanding of time representation on analog clocks, misplaced clock hands when drawing, and reversed or incorrect notation when reading times (e.g., 12:6 for 06:00).

The notation errors (like writing 12:6 or 6:2) suggest confusion between the hour and minute hands, and possibly a lack of understanding of how analog time maps to digital time formats.

In the elapsed time question, the student incorrectly calculated 07:00 to 09:00 as 9 hours, which points to a misconception about the concept of duration a common challenge when transitioning from concrete experiences to abstract measurement.

**3.11. Ontogenic Obstacles**

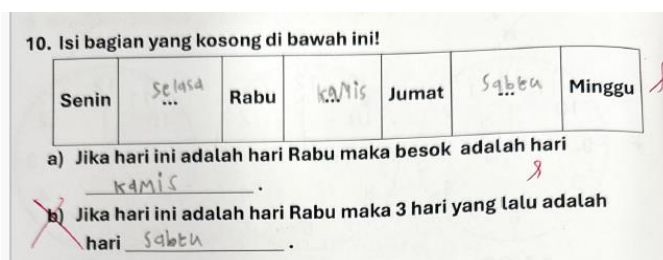
The student appears to struggle with abstract reasoning and symbolic interpretation, especially in coordinating the two clock hands and understanding their meaning.

This is typical of early-grade learners who are still developing temporal concepts and dual-representation reasoning (i.e., interpreting a single visual display in two ways: hours and minutes).

**3.12. Didactical Obstacles**

The errors may also reflect instructional gaps in differentiating the long hand (minutes) and short hand (hours), consistently linking analog and digital time formats, and practicing elapsed time problems with visual supports or number lines.

Based on Figure 11, here is an academic-style analysis of Student 1's response to Question 10, using the learning obstacles framework. Students were asked to complete a sequence of the seven days of the week and identify the next day and the day three days before a given day (Rabu/Wednesday).



**Figure 11** Student's Response to Question 10: Understanding Days of the Week.

From question 10 and Table 5, it can be identified that:

**Table 5** Students' Responses and Accuracy for Question 10.

Item	Expected Answer	Student's Answer	Correct?
a) If today is Wednesday, then tomorrow is...?	Kamis (Thursday)	Kamis (Thursday)	Correct
b) If today is Wednesday, what day was it three days ago?	Minggu (Sunday)	Sabtu (Saturday)	Incorrect

**3.13. Epistemological Obstacles**

The student demonstrates partial understanding of temporal sequencing correctly identifies the day after Wednesday (Kamis), but miscalculates the day three days before Wednesday (answers Sabtu instead of Minggu),

This suggests difficulty applying backward counting over a time sequence, a common challenge in early mathematical and logical reasoning.



### 3.14. Ontogenic Obstacles

The error in part (b) likely stems from a developmental challenge in abstract time reasoning. While counting forward is often easier for young learners, counting backward across week boundaries (e.g., from Wednesday to Sunday) requires a higher cognitive load and memory tracking. The student may have counted incorrectly as: Rabu → Selasa (1), Senin (2), Sabtu (3) → which skips Minggu, showing possible confusion with the order of the week.

### 3.15. Didactical Obstacles

Instruction may not have emphasized bidirectional reasoning with days of the week (i.e., "before" as well as "after"). Students may have had more exposure to routine sequences (e.g., "today is..., tomorrow is...") but less experience with multi-day intervals, particularly in reverse.

The analysis of students' performance reveals a range of learning obstacles that span multiple mathematical domains, including number operations, visual data interpretation, geometric reasoning, time concepts, and sequencing. While the student demonstrated strong conceptual understanding in areas such as shape recognition and forward day sequencing, consistent difficulties were observed in more complex cognitive tasks, particularly in interpreting pictograms, ordering 3D objects, working with analog clocks, solving word problems, and reasoning backward through time.

These difficulties indicate the presence of epistemological obstacles (misconceptions about mathematical concepts), ontogenic limitations (developmental readiness in abstract thinking and spatial reasoning), and didactical gaps (insufficient scaffolding or varied exposure in instruction). To address these challenges, targeted and differentiated instructional strategies are essential.

Recommendations include: strengthening foundational number sense through manipulatives and visual models; implementing guided CPA-based instruction for word problems; and explicitly teaching students to recognize key terms in problem contexts. For visual data interpretation, the use of pictogram activities with one-to-one matching and comparative questioning is advised. In spatial reasoning tasks, exposure to physical 3D objects and structured volume comparisons is crucial. To support time-related learning, educators should utilize physical clock models, timeline visuals, and real-life scheduling tasks. Additionally, the use of visual calendars, songs, and scaffolded routines can reinforce students' understanding of temporal sequencing both forward and backward. By aligning instructional methods with students' cognitive development and learning profiles, teachers can more effectively anticipate and overcome the learning obstacles that hinder early mathematical understanding.

Learning obstacles in early mathematics can be broadly categorized into epistemological, ontogenic, and didactical dimensions (Brousseau, 1997; Suryadi, 2021). *Epistemological obstacles* arise from students' conceptual misunderstandings or alternative frameworks for interpreting mathematical ideas (Radford, 2008). *Ontogenic obstacles* reflect developmental limitations, particularly in areas that require abstract reasoning, spatial visualization, or working memory, which are still maturing in early primary school learners (Clements & Sarama, 2009). Meanwhile, *didactical obstacles* emerge when instructional designs, task formats, or materials do not adequately scaffold students' thinking or connect with their prior knowledge (Prediger & Hefendehl-Hebeker, 2016).

The Concrete–Pictorial–Abstract (CPA) model, grounded in Bruner's theory of representation (1966), has been widely recommended for addressing such learning gaps, especially in number operations and word problems (Carpenter et al., 1999). Manipulatives and visual models serve as effective tools for building foundational number sense and supporting transitions from concrete to abstract mathematical thinking (Fuson et al., 2015; Moyer, 2001).

In geometry and data representation, visual-spatial reasoning is crucial. Young learners benefit from activities involving pictograms, object comparisons, and 3D shapes, which help develop skills in classification, estimation, and measurement (Clements & Sarama, 2004; NCTM, 2000). Similarly, time-related concepts such as telling time using analog clocks and understanding elapsed time require structured learning through real-life contexts, timelines, and physical clock models (Van de Walle et al., 2014).

Supporting early learners' understanding of temporal sequencing (e.g., days of the week, before/after reasoning) also requires integration of visual calendars, songs, and repetitive routines that align with students' cognitive profiles and lived experiences (Baroody & Dowker, 2003). Differentiated instruction that considers students' developmental stages can significantly reduce the impact of these learning obstacles and promote meaningful mathematical understanding.

### 3.16. Description of the Relationship Between Student Work and Teacher Interview Findings

The learning obstacles that arise are due to lack of time management, lack of focus, boredom, environmental conditions, internet network, limited wifi/internet quota and others (Listiani & Saragih, 2022). There is a strong correlation between the findings from students' work and the teachers' statements obtained through in-depth interviews. This alignment strengthens the validity of the identified learning obstacles experienced by Grade 1 students in mathematics instruction using CPA-approach. Moreover, in mathematics education research, combining student performance analysis with teacher insights

provides a deeper understanding of how instructional strategies affect conceptual development and where specific obstacles rise (Cobb et al., 2003). The following are the key connections observed:

### *3.17. Errors in Addition and Subtraction Concepts*

Student work revealed numerous mistakes in solving basic operations, especially subtraction and word problems. Teachers reported that students often confuse addition and subtraction, particularly when presented in abstract forms or within contextual problems. They also noted that the transition from concrete to abstract representations is not yet fully understood by many students. Furthermore, such confusion and misapplication of operations are typical examples of epistemological obstacles, where students construct alternative understandings of mathematical ideas (Radford, 2008), and ontogenic obstacles, which reflect developmental limitations in abstraction and symbolic reasoning (Clements & Sarama, 2009). Students' errors reflect the challenges described by teachers, specifically related to epistemological and ontological obstacles.

### *3.18. Difficulty Understanding Instructions*

Student responses showed misinterpretations of instructions, such as drawing when only an "X" mark was required. Teachers explained that many students still have developing reading skills, which leads to frequent misinterpretation of question prompts. This indicates the presence of didactical obstacles, where the difficulty lies not only in content understanding but also in interpreting how tasks are presented.

### *3.19. Concept of Time and Telling Time*

Student work demonstrated widespread errors in time-related questions, including incorrect placement of clock hands. Teachers identified time as one of the most challenging concepts, due to its abstract nature and symbolic interpretation. These difficulties align with epistemological and ontological obstacles, highlighting the need for stronger support at the concrete and pictorial stages of the CPA approach.

### *3.20. Use of Concrete Media and Visual Activities*

Teachers emphasized the importance of teaching aids such as toy clocks, 3D shape nets, and real-life objects to support understanding. Students who succeeded in tasks involving data grouping or visual components demonstrated that the use of concrete media positively impacted their comprehension. This confirms the effectiveness of the CPA approach when implemented consistently and creatively in classroom settings.

### *3.21. Disparities in Student Abilities*

Teachers noted significant differences in students' abilities, particularly between those who receive additional tutoring and those who do not. Student work also showed noticeable variation, with some performing well and others making frequent errors. This underscores the importance of adopting adaptive instructional strategies to accommodate the diverse backgrounds and learning needs of students.

Student work strongly supports and reflects the findings from teacher interviews. The identified learning obstacles whether cognitive, affective, or psychomotor consistently across both data sources. This demonstrates that a comprehensive understanding of learning obstacles can be achieved by integrating student performance analysis with teacher insights. In addition, students can also be given a lot of practice so that they get used to working on the questions given by the teacher (Listiani et al., 2019).

## **4. Conclusions**

This study examined the learning obstacles encountered by Grade 1 elementary students in understanding basic mathematical concepts through the use of a CPA (Concrete–Pictorial–Abstract)-based textbook approach. The analysis revealed three primary types of obstacles, such as Ontogenic Obstacles: Related to students' cognitive developmental readiness. Some students were unable to connect concrete experiences to abstract symbols, particularly in number operations and place value understanding. Didactical Obstacles: Emerging from instructional design or media that do not adequately support students' gradual transition from concrete to abstract thinking. The CPA-based textbook used in this study was found to be suboptimal in facilitating this progression. Epistemological Obstacles: Linked to how students interpret and internalize mathematical concepts. Many students tended to rely on direct counting strategies without a solid grasp of the underlying principles of addition and subtraction. The findings highlight the importance of selecting instructional approaches that align with students' cognitive development stages. The CPA model, while pedagogically sound, needs to be implemented more systematically to effectively support conceptual understanding. Teachers must also be aware of students' varying capabilities and limitations to anticipate and address learning obstacles more effectively.

The limitations of the study are all tasks were administered in a single session. According to teacher feedback, the number of analytical questions made students feel bored and fatigued during the activity. Instructions and sample questions should have been provided prior to the main tasks to help students better understand expectations. Instruction on the concept of time was insufficiently developed, resulting in many students struggling to determine or represent time accurately. Approximately 10 out of 20 students received private tutoring, which allowed them to complete tasks more quickly, while the remaining students required additional assistance. Many students had reading difficulties, which affected their performance on word problems this concern was also raised by teachers during interviews. In addition and subtraction tasks, most students showed basic procedural ability, but often made careless errors due to a lack of attention to detail.

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### Ethical Considerations

This study was conducted in compliance with ethical standards for research involving human participants, particularly young children. Ethical approval for the research was obtained from the Research and Community Service Institute (LPPM), Universitas Pelita Harapan, under the internal research grant project number P-075-FIP/XII/2024. Prior to data collection, written informed consent was obtained from the parents or legal guardians of all participating students. In addition, permission and cooperation were secured from the principal and classroom teachers of the participating school. To ensure participants' rights and well-being were protected: Voluntary participation – Students and teachers participated on a voluntary basis, and they were free to withdraw from the study at any time without penalty. Confidentiality – All data were anonymized; participants' names and identifying information were removed from transcripts, observation notes, and student work samples. Child protection – Research activities were conducted during regular school hours under teacher supervision to ensure a safe learning environment. Data security – All research data were stored securely and used solely for academic purposes.

### Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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### References

- Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, 14(2), 243–248. <https://doi.org/10.3758/BF03194059>
- Babchuk, W. A. (2017). Book Review: Qualitative Research: A Guide to Design and Implementation (4th ed.), by S. B. Merriam and E. J. Tisdell. *Adult Education Quarterly*, 67(1), 71–73. <https://doi.org/10.1177/0741713616671930>
- Baroody, A. J., & Dowker, A. (2003). *The Development of Arithmetic Concepts and Skills: Constructing Adaptive Expertise*. Lawrence Erlbaum Associates.
- Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative Research Journal*, 9(2), 27–40. <https://doi.org/https://doi.org/10.3316/QRJ0902027>
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp0630a>
- Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. Kluwer Academic Publishers.
- Bruner, J. S. (1996). *The culture of education*. Harvard University Press.
- Cai, J., Kaiser, G., Perry, B., & Wong, N. . (2020). *Compendium for early career researchers in mathematics education*. Springer.
- Clements, D. ., & Sarama, J. (2004). *Hypothetical Learning Trajectories: A Special Issue of Mathematical Thinking and Learning (1st ed.)*. Routledge. <https://doi.org/10.4324/9780203063279>
- Clements, D. H., & Sarama, J. (2009). *Learning and Teaching Early Math: The Learning Trajectories Approach*. Routledge.
- Cobb, Paul, Confrey, Jere, diSessa, Andrea, Lehrer, Richard, & Schauble, Leona. (2003). Design Experiments in Educational Research. *Educational Researcher*, 32(1), 9–13. <https://doi.org/10.3102/0013189X032001009>
- Creswell, J. W. (2017). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches (5th ed.)*. CA: SAGE Publications.
- Denzin, N. K. (1978). *The research act: A theoretical introduction to sociological methods (2nd ed.)*. McGraw-Hill.
- Fuson, K. C., Clements, D. H., & Sarama, J. (2015). Making early math education work for all childrenmaking early math education. *Phi Delta Kappan*, 97(3), 63–68. <https://doi.org/10.1177/0031721715614831>

- Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. SAGE Publications.
- Listiani, T., Dirgantoro, K. P. S., Saragih, M. J., & Tamba, K. P. (2019). Analisis Kesalahan Mahasiswa Pendidikan Matematika Dalam Menyelesaikan Soal Geometri Pada Topik Bangun Ruang [Error Analysis of Students in the Mathematics Department in Solving Geometry Problems on the Topic of Solid Figures]. *JOHME: Journal of Holistic Mathematics Education*, 3(1), 44. <https://doi.org/10.19166/johme.v3i1.1708>
- Listiani, T., & Saragih, M. J. (2022). Hambatan Belajar Mahasiswa dalam Pembelajaran Jarak Jauh pada Mata Kuliah Geometri Analitik. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, 06(02), 2201–2211.
- Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and implementation (4th ed.)*. Jossey-Bass.
- Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook (3rd ed.)*. SAGE Publications.
- Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, 47(2), 175–197. <https://doi.org/10.1023/A:1014596316942>
- NCTM. (2000). *Principles and Standar dor School Mathematics*. The National Council of Teachers of Mathematics.
- Patton, M. Q. (2015a). *Qualitative research & evaluation methods: Integrating theory and practice (4th ed.)*. Sage Publications.
- Patton, M. Q. (2015b). *Qualitative research & evaluation methods (4th ed.)*. SAGE Publications.
- Piaget, J. (1952). *The Child's Conception of Number*. Routledge & Kegan Paul.
- Prediger, S., & Hefendehl-Hebeker, L. (2016). On the Relevance of Epistemological Awareness for Classroom Practices of Teachers. *Journal Fur Mathematik-Didaktik*, 37(1), 239–262. <https://doi.org/10.1007/s13138-016-0085-5>
- Radford, L. (2008). *The ethics of being and knowing: Towards a cultural theory of learning*. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in Mathematics Education*. Sense Publishers.
- Suryadi, A. (2021). *Diagnosis kesulitan belajar matematika SD*. Refika Aditama.
- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2014). *Elementary and Middle School Mathematics: Teaching Developmentally (9th ed.)*. Pearson.
- Yin, R. K. (2018). *Case study research and applications: Design and methods (6th ed.)*. Sage Publications.