

Analogical reasoning in algebraic learning within mathematics education: A bibliometric study



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Abstract This study aims to observe the development of research trends in analogical reasoning on algebra topics using a bibliometric approach and analyze its role in learning. This study uses a systematic literature review (SLR) with a bibliometric approach, with data obtained from the Scopus database during the period 2011–2025 and visualized by VosViewer. This study is driven by the results of previous studies that state that analogical reasoning plays an important role as a cognitive process in understanding algebraic structures and reducing students' misconceptions in learning algebra. The results of the analysis conducted show that in 2024, research on the topic of analogical reasoning and algebra increased significantly, with a total of 35 articles. The most contributing author is Christian Antić from Austria and the United States, as the country that contributes the most to this topic. CNRS is also the top institution and the most advanced in Mathematics (Q1), as the journal that contributes the most. This study also shows that analogical reasoning plays an important role in overcoming students' errors related to the symbolic structure of algebra and provides a conceptual framework for strengthening students' mathematical understanding. The uniqueness of this study lies in mapping global research trends and identifying gaps in the study of analogical reasoning in universal algebra, which remains underexplored. The results of this study contribute to providing a more in-depth picture of the development and role of analogical reasoning as a conceptual approach in strengthening students' mathematical abilities and confirm that analogical reasoning has the potential to be a learning method and strategy that can be applied in mathematics learning activities, especially in algebra learning.

Keywords: algebra, analogy, literature reviews, scopus, universal algebra

1. Introduction

Reasoning plays an important role in improving students' higher-order thinking skills, which are needed to address 21st-century challenges (Mukuka et al., 2023). Reasoning is also a cognitive aspect that students must develop and possess during the learning process to develop mathematical abilities (Kliziene et al., 2022). Reasoning not only supports the problem-solving process but also plays a important role in forming concepts and generalizing more complex mathematical ideas.

Reasoning plays a important role in a person's thinking process (Rivas et al., 2022). Reasoning is a form of thinking that interprets a statement based on statements whose truth has been previously proven (Yerizon et al., 2023). Research conducted by Raj et al. (2022) emphasized that reasoning skills play a crucial role in solving various problems, so this ability must be developed and practiced by students. This view is very much in line with the findings of Hačarjana and Namsone in their 2024 study, which also emphasized that reasoning is one of the key competencies that must be possessed and mastered by students (Hačarjana & Namsone, 2024).

As Yu et al. (2024) stated, it is closely related to problem-solving; therefore, reasoning will most often be a part of the process. Then, the ability to understand and apply apparent similarities between two different situations will involve a skill, namely reasoning (Gentner & Smith, 2013). This is also supported by the statement of Gray & Holyoak (2021) that how well students solve any problem depends largely on their reasoning skills. Even if there is a difficult problem and students use an analogy with a problem they had already faced, finding the solution will be much easier (Chiu et al., 2022).

Analogies in learning algebra enable one to create correspondences between the concrete and symbolic and establish solutions relating them structurally (Theba et al., 2024). Various research findings suggest that students still make a lot of mistakes and misconceptions in understanding the learning of algebra (Rafiepour et al., 2023; Stemele & Jina Asvat, 2024; AL-Rababaha et al., 2020). For instance, students tend to commit errors in transforming algebraic expressions due to their inability to use signs, priorities of operations, or the meaning of letters/symbols (Vlassis & Demonty, 2022). According to studies conducted by Stemele and Jina Asvat (2024) some common errors that students make are failure of understanding variables, negative signs, and context problem in symbolic form. Further, students typically write additional entries in the expression that do not carry over correctly between parentheses or grasp more intricate structures of algebra (Theba et al., 2024). Consequently, analogical reasoning could be considered as an instructive technique to develop the understanding of abstract concepts in algebra and minimize procedural mistakes.



Analogical reasoning in algebra learning also helps in reducing misconceptions while enhancing students' representational skills. Recent research supports the fact that analogical approaches strengthen conceptual connections between familiar and newly learned concepts (Tise et al., 2023; Gray & Holyoak, 2021). Through analogies, students are able to compare similarities and differences between problem structures, leading to deepened conceptual understanding (Wang, 2022).

Although many studies have discussed analogical reasoning in mathematics education, very few have specifically examined this topic using a bibliometric approach. In particular, Research on how analogical reasoning is applied in algebra, especially in universal algebra, remains very limited. Most previous studies have focused only on student learning outcomes or on how analogical reasoning is used in learning, without mapping the overall trends and developments in this topic. Therefore, this study was conducted to fill this gap by reviewing and analyzing Research patterns, main themes, and future opportunities related to analogical reasoning in algebra learning. Therefore, this study aims to fill this gap by reviewing and analyzing Research patterns, key keywords, and future opportunities related to analogical reasoning in algebra learning, with a specific focus on universal algebra.

2. Research Method

2.1. Types of research

This research was conducted using bibliometric analysis with the SLR (Systematic Literature Review) method, which aims to collect, initiate, and present data related to analogical reasoning in algebra (Fitroni et al., 2025). This was also done to provide a complete picture of the developments, approaches, and trends in the field of analogical reasoning related to algebraic problem solving.

Bibliometric analysis is used to investigate research findings, keywords, developments, trends, and scientific performance. This study applies a five-stage bibliometric approach as outlined in Figure 1. Firstly, Definition of Keywords: In this stage, research questions and objectives are set, and clear definitions of search terms are established, along with boundaries such as year, language, field, and document type. The second stage is Initial Search Results, which utilizes international articles indexed by Scopus to obtain several initial articles. After that, Search Refinement is used to clean the data by sorting and removing data unrelated to the topic of the research. Cleaning the data involves eliminating inconsistencies, double-checking keyword consistency, and eliminating duplication in order to organize the data in a better way. The fourth stage is the collection of data from the database, which aims at processing important information such as titles and keywords, the abstract of the research work, the research field, and the number of citations. Finally, to assess the progress in mapping these relationships, data analysis is conducted in terms of the relationships between authors, sources, topics, research fields, and keywords with the aim of identifying the key themes and developments within the topic (Öztürk et al., 2024; Donthu et al., 2021; Passas, 2024; Lintangesukmanjaya et al., 2025).

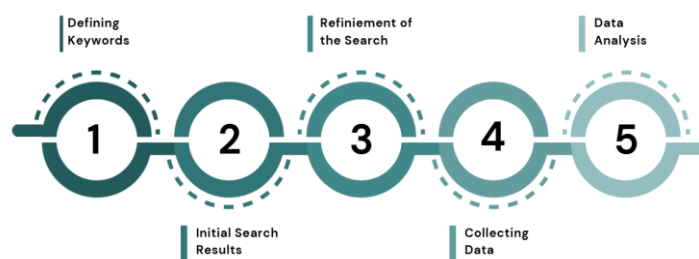


Figure 1 Research design.

2.2. Instruments and data processing

In processing bibliometric data, the SLR instrument is used to measure, visualize, and observe the distribution map and relationships between keywords, assisted by VosViewer software. In this study, the data population used comes from international articles indexed by Scopus in the period 2011 - 2025, which were obtained online on October 13, 2025. After that, the data obtained was analyzed according to the stages of the SLR method, namely using the PRISMA design, which aims to assist in compiling structured and transparent reports about the inclusion and exclusion of articles in each review (Kaur et al., 2024).

The PRISMA design shown in Figure 2 aims to structure and report the literature review and meta-analysis process transparently. This design is used to assist in collecting, identifying sources, and filtering based on include and exclude criteria, ensuring that each stage of the literature review process is carried out consistently until the final topic is determined. This design is also used to select topics and facilitates readers in assessing the completeness of the stages, reducing the potential for errors, as depicted in a flowchart (Page et al., 2021). The data will be analyzed for developments, journals, authors,

affiliations, countries, and keywords for each article. The results of this analysis will be generalized to yield findings on analogical reasoning in algebraic topics over the past 15 years.

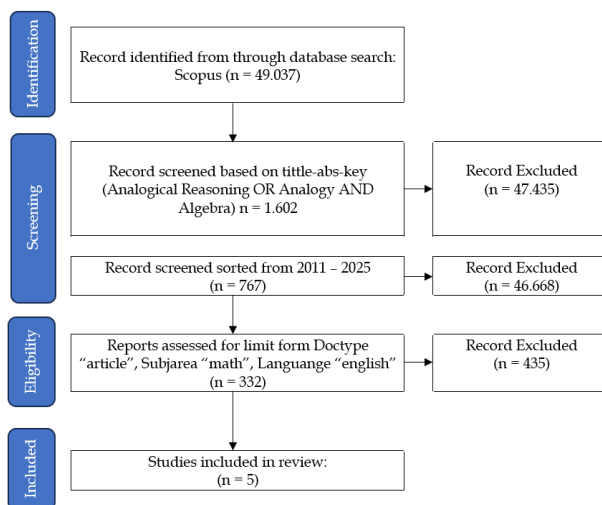


Figure 2 PRISMA chart.

Figure 2 shows that, with the keywords "Analogical reasoning OR Analogy AND Algebra," 1,602 were obtained. After filtering from several categories, namely the period from 2011 - 2015, Doctype "article", Subjarea "math", and Language "english", obtained as many as 332 documents. Of the 332 documents on universal algebra, only 5 articles are accessible. Therefore, the number of articles reviewed in the literature review is 5.

3. Results

3.1. Growth trends and general information

The database used in this study was sourced from Scopus for the period 2011–2025 (the last 15 years). This is because many articles related to analogical and algebraic reasoning have been published over the past 15 years, making it sufficient for the analysis. Figure 3 shows general information from 332 articles from 2011 to 2025.

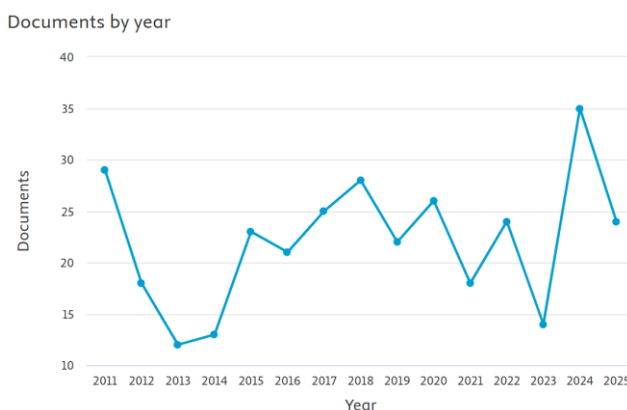


Figure 3 Document per year. Source: Scopus database.

The analysis results of the PRISMA design show that many publications with the keywords "Analogical Reasoning" OR "Analogy" AND "Algebra" yielded 767 relevant articles in the period 2011 - 2025. The next stage was filtering by several categories: Doctype "article", Subjarea "math", and Language "english", resulting in 332 articles selected for further analysis, as shown in Figure 2. The development of these articles over the years is shown in Figure 3. The figure shows a trend of increasing publications on analogical reasoning in algebra. The largest increase occurred in 2023-2024, with the number of publications reaching 35 in 2024. Meanwhile, in 2025, the number of publications decreased, as data collected through October recorded only 24 documents. This pattern of increasing publications shows that the topic of analogical reasoning in algebra has attracted greater attention from researchers in recent years.

3.2. Contribution by journals



On the topic of analogical and algebraic reasoning, 160 journals contributed. Figure 4 shows the distribution of the top five Scopus-indexed journals that have published articles related to analogical and algebraic reasoning.

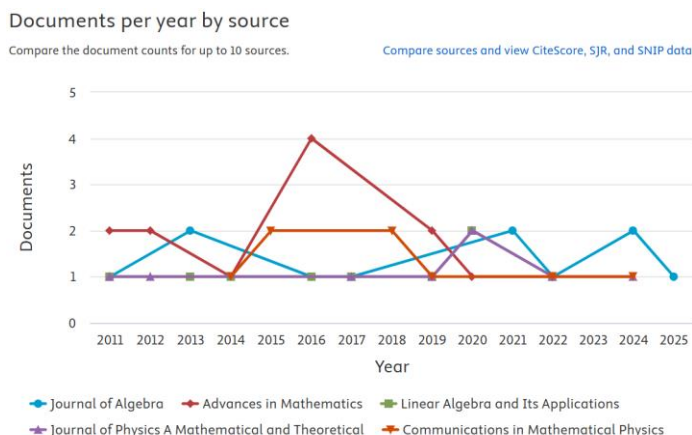


Figure 4 Top five journals in scopus. Source: Scopus database.

Figure 4 shows that the Advances in Mathematics Journal (Q1) is the journal that publishes the most articles on related topics with 13 articles. Followed by the Journal of Algebra (Q2) with 11 published articles, the Journal of Linear Algebra and its Applications (Q1) with nine published articles, the Journal of Communications in Mathematical Physics (Q1) with eight published articles, and the Journal of Physics A: Mathematical and Theoretical (Q1) with eight published articles. In addition to the five journals with the most publications, several others also publish articles on this topic. These five journals can also be a reference for publishing articles on the topic of analogical and algebraic reasoning. The growth of publications of articles published in these journals from 2011 to 2025 can be seen in Table 1.

Table 1 Top five journals.

Source Title	Cite Score	Highest Percentile	% Cited	SJR	Publisher	The number of article
Advances in Mathematics	3.0	84% (Q1)	68	2.094	Elsevier	13
Journal of Algebra	1.6	63% (Q2)	54	1.029	Elsevier	11
Linear Algebra and its Applications	2.2	82% (Q1)	60	0.997	Elsevier	9
Communications in Mathematical Physics	4.9	90% (Q1)	76	1.506	Springer Nature	8
Journal of Physics A Mathematical Theoretical	3.8	82% (Q1)	70	0.659	Institute of Physics Publishing	8

Source: Scopus database.

Table 1 shows that of the five journals, four have a percentile range of 75% - 99%, meaning they are in the first quartile (Q1). Meanwhile, the Journal of Algebra has a percentile of 63%, indicating it is in the second quartile (Q2). Furthermore, the SJR of all five journals is above 0.5. Therefore, these five journals are excellent for publishing articles. Figure 4 also shows the development of articles published by the journals over the past 15 years.

3.3. Contributions by author

There are 160 authors from various countries who have contributed to published articles on the topic of analogical and algebraic reasoning. Figure 5 shows the distribution of the 10 authors with the most citations on the topic of analogical and algebraic reasoning.

Based on Figure 10, the most contributing authors are Antić, C. from Technische Universität Wien, Vienna, Austria, with a total of 4 published articles related to the topic of analogical and algebraic reasoning. The focus of Antić, C.'s research topic is on analogical reasoning. Followed by Bodaghi, A., from Islamic Azad University, Tehran, Iran, who also has four published articles related to analogical and algebraic reasoning. Next, Campbell, J.M. from Dalhousie University, Halifax, Canada, Ciaglia, F.M. from Universidad Carlos III de Madrid, Getafe, Spain, and Hicks, M.D. from Virginia Polytechnic Institute and State University, Blacksburg, United States, all of whom have three published articles. Arnind, J. from Linköpings Universitet, Linköping, Sweden, Belluce, L.P. from The University of British Columbia, Vancouver, Canada, Carlen, E.A. from the Department of Mathematics, Piscataway, United States, Chimoni, M. from the University of Cyprus, Nicosia, Cyprus, and Christou, C. from the University of Cyprus, Nicosia, Cyprus, each have two published articles on related topics. This shows that the 10 authors have made significant contributions to research on the topic of analogical and algebraic reasoning.



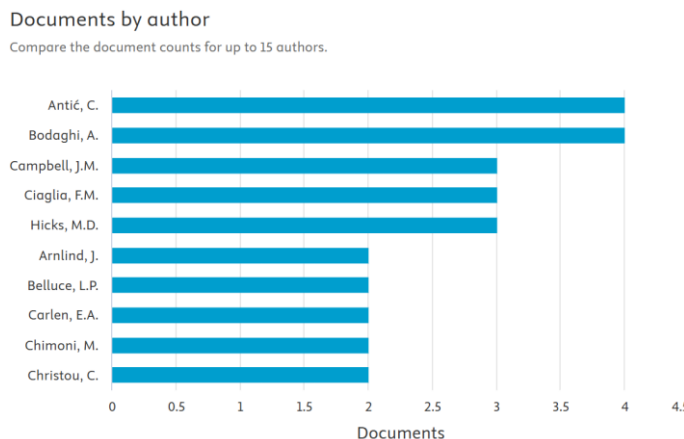


Figure 5 Top ten authors. Source: Scopus database.

3.4. Subtopic contribution by institutions

A total of 160 institutions from various countries have been involved in publications related to the topic of analogical and algebraic reasoning. Figure 6 shows the distribution of articles based on the top institutions that published the most articles on the topic of analogical and algebraic reasoning.

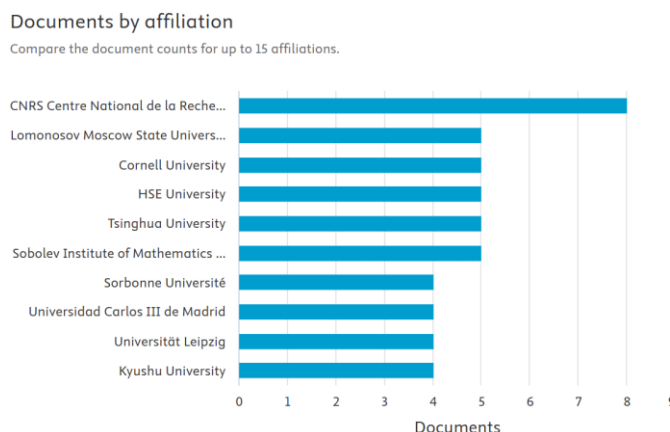


Figure 6 Top ten institution. Source: Scopus database.

The ten institutions contributing to this topic are the CNRS Centre National de la Recherche Scientifique with eight publications. Lomonosov Moscow State University, Cornell University, HSE University, Tsinghua University, and the Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences each have five publications. Sorbonne Université, Universidad Carlos III de Madrid, Universität Leipzig, and Kyushu University each have four publications.

3.5. Distribution by country

On the topic of analogical and algebraic reasoning, 58 countries contributed. Figure 7 shows the distribution of articles across the top 10 countries that published the most articles in research related to the topic of analogical and algebraic reasoning.

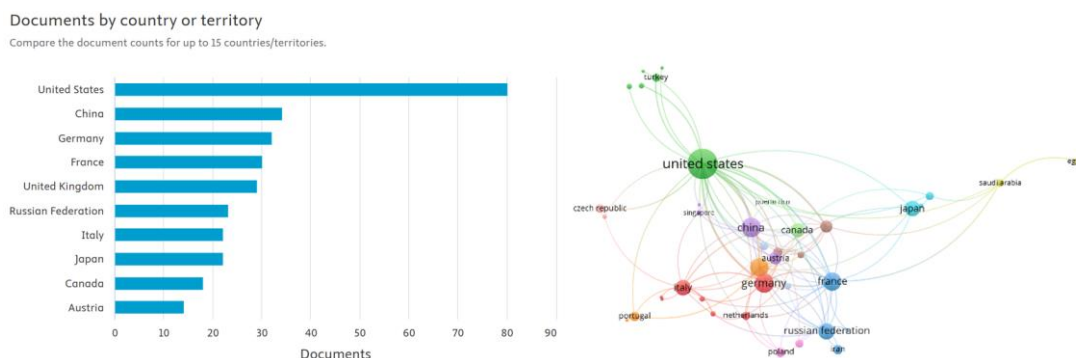


Figure 7 Distribution of articles by country. Source: Scopus database.



Based on Figure 7, the country with the largest contribution to the publication of articles on the topic of analogical and algebraic reasoning is the United States, with 80 articles published. On the topic of analogical reasoning, the United States is the most dominant and productive institution in publishing articles, especially at Cornell University, which also ranks second in terms of the number of articles published on this topic. Therefore, American authors have conducted extensive research on analogical and algebraic reasoning. According to Gökçe and Güner (2021) and Suseelan et al. (2022), their research also shows that the United States is the most active country in publishing scientific articles. Therefore, future researchers can use the United States as a reference in research on analogical reasoning in algebra topics. The second country contributing to this topic is China, with 34 articles. Followed by Germany with 32 articles, France with 30 articles, the United Kingdom with 29 articles, the Russian Federation with 23 articles, Italy with 22 articles, Japan with 22 articles, Canada with 18 articles, and Austria with 14 articles.

3.6. Keywords analysis

Based on the mapping results of the keywords “Analogical Reasoning OR Analogy AND Algebra,” the most frequently appearing word is Matrix Algebra. Matrix Algebra appears 35 times in articles published from 2011 to 2025. In addition, the word algebra also appears 29 times, eigenvalues and eigenfunctions appear 10 times, linear algebra appears 9 times, and polynomials appear 7 times. Furthermore, the visualization of the relationship between the keywords that appear can be seen in Figure 8.

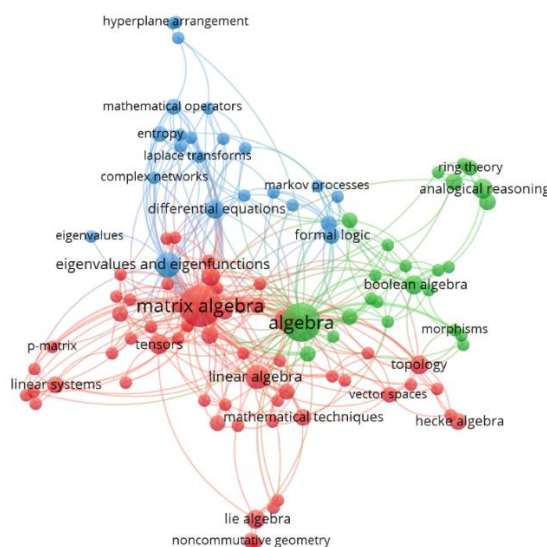


Figure 8 Top Network visualization.

This topic has three clusters: the red cluster contains 56 items, with the most significant occurrence being matrix algebra. The green cluster contains 24 items, with the most occurrences being algebra. The blue cluster contains 23 items, with the most occurrences being eigenvalues and eigenfunctions. Furthermore, the results of keyword mapping supporting analogical and algebraic reasoning are grouped into the top 10 keywords based on their distribution in Table 2.

Table 2 Top ten keywords.

Rank	Keywords	Total Link Strength
1.	Matrix algebra	111
2.	Algebra	67
3.	Eigenvalues and eigenfunctions	40
4.	Linear Algebra	20
5.	Matrix	20
6.	Tensors	19
7.	Polynomial	18
8.	Topology	18
9.	Differential equations	17
10.	Formal logic	17

Source: Scopus database

The top 10 keywords most significantly determine the emergence of the main topic of interest to researchers related to the research. Meanwhile, keywords with fewer occurrences indicate that research with those keywords is still limited.



Therefore, to identify research novelties in the topic of analogical and algebraic reasoning, the research will be conducted using the emergence visualization in Figure 9.

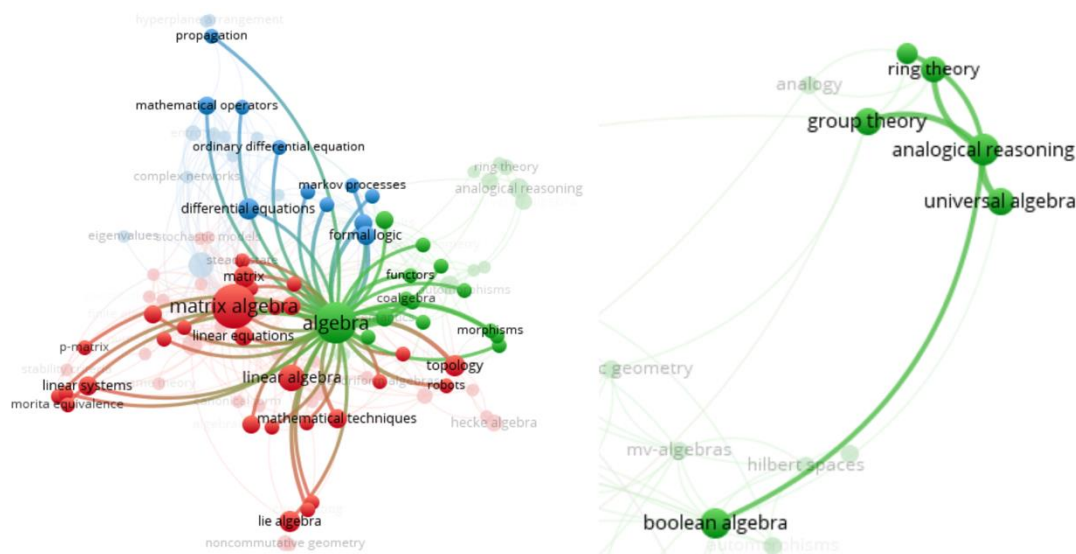


Figure 9 Network Visualization with algebra keywords and analogical reasoning keywords.

Based on Figure 9, algebra is related to several research keywords in other clusters, such as algebraic research on mathematical operators and formal logic (blue cluster). Furthermore, algebraic research is also related to mathematical techniques, matrix algebra, linear algebra, and linear equations (red cluster). In the green cluster, algebraic research is also related to functors. Keywords directly related to algebra indicate that there have been publications related to these topics. Based on Figure 9, analogical reasoning is only related to a few keywords, namely universal algebra, Boolean algebra, group theory, ring theory, and abstract algebra. Therefore, research related to analogical reasoning in this algebraic topic is still little discussed and can be followed up by future researchers.

There are several topics that have been widely discussed by researchers. However, there are also many topics that have received little research. Therefore, there is still an opportunity to further explore these less-explored topics. Figure 10 shows an overly visualization and density of some of these topics.

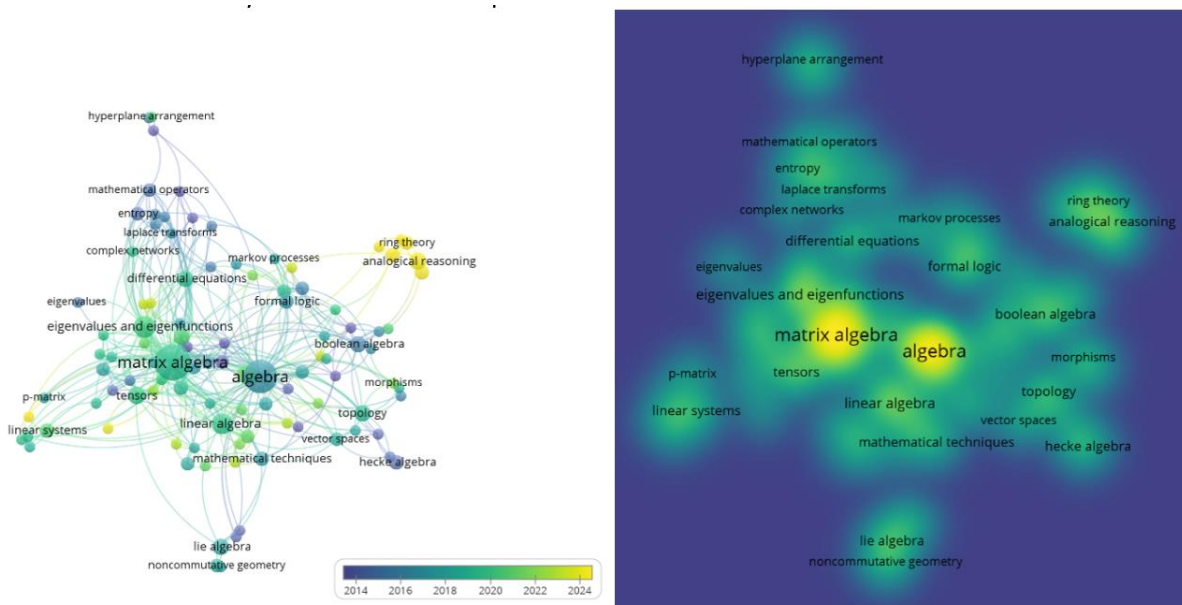


Figure 10 Overly and density visualization.

Based on Figure 10, lighter colors indicate new research, while darker colors indicate that the research has not been studied for a long time. Furthermore, Figure 10 shows that analogical reasoning related to universal algebra, abstract algebra, group theory, and ring theory is trending. This indicates the extent of research conducted. Meanwhile, the research related to algebra in the figure indicates that it has not been studied for a long time.

Figure 10 shows that the brighter the research topic, the more research has been conducted, and the darker the research topic, the less research has been conducted. For example, algebra is brighter than mathematical operators. This indicates that there is more research on algebra than on mathematical operators. Thus, the dark theme shown in Figure 10 provides an opportunity for researchers to conduct further research. Keywords related to algebra that have not been widely studied are mathematical operators, linear algebra, mathematical techniques, formal logic, and differential equations. On the topic of analogical reasoning, there is also little discussion of universal algebra, so further research is needed.

Research on analogical reasoning is still limited. Analogical reasoning is also a growing trend in 2024. Research on analogical reasoning is only concerned with one form of algebra, namely universal algebra. Therefore, research on analogical reasoning in universal algebra needs to be further pursued, given its current trend. To refine the findings and recommendations, an analysis of articles related to analogical reasoning and universal algebra is needed, as listed in Table 3.

Table 3 Analyze related articles.

Title	Author(s)	Citation; SJR	Research Findings and Recommendations
Analogical proportions	Christian Antić	0.419; Q3	This study found that analogical propositions can be formalized within a universal algebraic framework with logical properties such as symmetry and reflexivity, demonstrating the relevance of analogical reasoning in understanding mathematical relationships. It is recommended that this model be used to develop learning strategies that emphasize relational relationships between algebraic concepts.
Analogical proportions in monounary algebras	Christian Antić	0.419; Q3	Outcomes of this research show that it is possible to explain analogous proportions in monounary algebra by numerical differences alone, reinforcing the importance of simple relations in knowing about relational equivalence. It is recommended that this concept be applied to students to identify equivalence patterns in algebraic manipulations.
Generalization-based similarity	Christian Antić	0.419; Q3	This work extends the analogy to generalization-based similarity and demonstrates the possibility of formally modeling structural similarities between algebraic elements. Further, it is recommended that this approach be adopted to train generalization and analogy reasoning skills in solving algebra problems.
“I’ll just try to mimic that”: an exploration of students’ analogical structure creation in abstract algebra	Michael D. Hicks	1.589; Q1	Findings showed that the way students constructed analogical structures in learning abstract algebra, within the explanation framework known as ARM, consisted of a variety of methods. Algebra instruction should be designed in a way that fosters productive analogical construction to strengthen students’ conceptual understanding.
Boolean Proportions	Christian Antić	0.419; Q3	This study demonstrates that analogical propositions in Boolean algebra satisfy most of the axioms of classical analogy, strengthening the validity of analogical models in the context of logic and symbolic structures. It is recommended that this approach be utilized to deepen students’ understanding of logical relations in algebraic expressions.

Source: Scopus database

Based on the analysis of the five articles in Table 3, it can be concluded that analogical reasoning in the context of algebra plays an important role as a cognitive mechanism for understanding equivalence, structure, and relational relationships between mathematical concepts. A study by Christian Antić showed that analogical propositions can be formalized mathematically in various algebraic systems, such as universal, monounary, and Boolean algebra, which confirms the existence of relational similarities and symmetries between algebraic elements. Meanwhile, research by Hicks highlighted the pedagogical aspect of analogical reasoning, namely, how students construct and utilize analogical structures to understand abstract concepts in algebra. Overall, these results show that analogical reasoning not only functions as a means of knowledge transfer but also as a reflective process that strengthens conceptual understanding and generalization abilities in algebra learning.

4. Discussion

This bibliometric analysis of analogical reasoning and algebra shows a variety of findings that assist in the development of research and practice in mathematics learning. The quantitative analysis shows an increase in publications between 2011 and 2025, with peaks in 2023 and 2024, indicating a growing number of studies on the application of analogical reasoning in



algebra. This corroborates studies showing that analogical reasoning is an important tool in bridging gaps in conceptual understanding of abstract algebraic structures (Hicks, 2024). From the quantitative analysis, *Advances in Mathematics (Q1)* is the journal publishing the most on this topic, with 13 articles. The most published author is Christian Antić from Austria, who has published four articles on this topic. The most contributing institution is CNRS, with eight articles. Finally, the country publishing the most on this topic is the United States of America, with 80 articles. This is well in line with other research studies (Sibgatullin et al., 2022; Thelwall & Sud, 2022).

The synthesis of the findings of several relevant articles reviewed from the perspective of analogical reasoning and universal algebra, as reviewed above, indicates that research on analogical proportions, generalization-based similarity, analogical structure creation, monounary algebra, and Boolean algebra provides a strong theoretical and empirical foundation in explaining how analogies can be used in such algebra. Antić (2022), through the study of Analogical Propositions, shows that the relational equivalence relationship of four elements that are reflective and symmetric can be understood as an analogical proposition. In the context of monounary algebra, Antić states that this relationship can be expressed through the same numerical difference in representing a very basic form of relational equivalence in algebraic structures. Boolean proportions also extend analogy to the realm of symbolic logic, which emphasizes that analogy is not only a cognitive process, but can also be formed in a formal logic-based system. Later, Antić's (2025) research on generalized-based similarity extended this perspective by showing that similarities between algebraic elements can be explained through generalizations, with analogy being a specific topic of the more familiar structural similarity. These findings indicate that analogical reasoning has a consistent mathematical basis and can be used as a reference in understanding conceptual similarities in algebraic operations (Cangiotti & Nappo, 2022; Chimoni et al., 2023).

Meanwhile, Hicks (2024) made the important pedagogical contribution of investigating the ways in which students create and utilize analogical structures (analogical structure creation) in understanding abstract algebraic concepts. His findings showed that such analogical processing involves connecting known structures to new ideas through some form of relational mapping mechanism. This further reinforces the earlier claim that analogies are imperative for any attempt by students to transfer more meaningful conceptual knowledge (Gray & Holyoak, 2021; Singer & Voica, 2022).

Antić & Hicks add that, in the context of learning, applying analogical reasoning nurtures deeper conceptual understanding. Analogy-based approaches have been found to improve students' ability to transfer understanding (Vula & Berisha, 2022; Lailiyah et al., 2022). More recently, Tise et al. (2023) discovered that analogies-based learning strategies that teach students to use analogies strengthen metacognition and higher-order thinking skills. Thus, applying the principles of analogical proportions, generalization-based similarity, and creation of an analogical structure is possibly one more strategy that could help learners overcome typical difficulties related to sign errors, variable understanding, and pattern generalization. Such findings extend the understanding that analogies have relevance beyond being a theoretical concern of mathematics, into the practices of school learning (Gray & Holyoak, 2021).

The two most important directions for development that can be synthesized from these research findings are, at a theoretical level, the use of analogical reasoning in explaining mathematically the equivalence and similarities of relations between structures in algebra and, practically, using analogical reasoning in learning algebra in order to build concepts and avoid common misconceptions, such as the use of parentheses, understanding variables, and relationships among operations.

Limitations of the study are that the database used is only sourced from Scopus without considering other sources. Research on analogical reasoning in universal algebra based on Scopus is still very scarce. The search results revealed no more than 6 articles, and only 5 could be accessed and analyzed in this study. The reasons few studies could be retrieved and analyzed lie in the lack of published materials linking analogical reasoning to universal algebra. Thus, the five articles that could be accessed and analyzed in this study represent all available ones relevant to Research on analogical reasoning in universal algebra. Several studies reviewed show that analogical reasoning plays an important role in helping students better understand algebraic concepts and symbols. Theoretically, analogical reasoning can be applied to various algebraic systems within the scope of universal algebra, such as proportionality algebra, Boolean algebra, monounary algebra, generalization-based similarity, and analogical structure creation. Pedagogically, this study can be a reference for educators and researchers to design analogy-based learning strategies that can overcome misconceptions about algebraic operations and variables. Bibliometric analysis also demonstrated that research on the topic of analogical reasoning and universal algebra is still focused on the conceptual aspect, while its application in the learning context is still limited. This opens opportunities for research in mathematics education to adapt theory into concrete learning strategies that fit the characteristics of students at various levels.

5. Future Perspectives

Recommendations for future Research on analogical reasoning in the context of algebra can be addressed in several directions. First, the results of this study open the door to expanding bibliometric exploration into other areas of mathematics, such as geometry, trigonometry, or statistics, to determine whether analogical reasoning exhibits similar Research trends across these fields. Secondly, the incorporation of analogical reasoning in the classroom setting requires further study, primarily through the design of models for learning through analogy that can improve understanding of algebraic functions. The Influence

of such models on students' problem-solving concepts and processes can be examined. Third, one challenge lies in linking theoretical findings in universal algebra with empirical studies in mathematics education, where students' cognitive processes can be observed in real-life learning environments.

Furthermore, future Research should consider using mixed-method or longitudinal approaches to capture the long-term effects of analogy-based learning. Ultimately, this direction will strengthen the theoretical and practical framework for utilizing analogical reasoning to enhance understanding in mathematics learning.

6. Final Considerations

The bibliometric analysis reveals a tendency of research on implementing analogical reasoning into solving algebraic problems, increasing during 2011–2025 and reaching its peak in 2023–2024, while the United States, China, and Germany dominated the research contribution. The key messages in this paper point out that, first, there are two salient orientations of analogical reasoning: the theoretical one, which may be formalized into a variety of algebraic systems, such as proportionality algebra, Boolean algebra, monounary algebra, generalization-based similarity, and the construction of analogical structures; and the pedagogical one, which may be applied to analogy-based learning strategies that have the potential to nurture conceptual understanding among students. Based on previous Research, this review suggests that analogical reasoning can be an effective way for students to connect their conceptual and procedural understanding, as well as their misconceptions about the symbolic structure of algebra. Further studies are recommended to develop analogy-based learning strategies and to investigate how analogical reasoning can be applied to learn algebraic topics, which are still rarely studied within the scope of universal algebra.

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7. Declarations

7.1. Ethical considerations

Not applicable.

7.2. Use of artificial intelligence (AI)

The authors declare that Grammarly's artificial intelligence (AI) tool was used exclusively for grammar correction and language editing. This use of AI did not influence the scientific content, Research design, data analysis, interpretation, results, or conclusions of the manuscript. Full responsibility for the content of the manuscript remains with the authors.

7.3. Conflict of interest

The authors declare no conflicts of interest.

7.4. Funding

This research did not receive any financial funding.

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