

How is collaborative argumentation in mathematics education? Systematic literature review



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Abstract This study aims to explore the definition, situations that require collaborative argumentation, characteristics of problem-solving tasks in collaborative argumentation studies, and framework used by researchers to analyze dialogs in collaborative argumentation. The researcher conducted a systematic literature review (SLR) from 2009 to 2024 on the Scopus database using the keywords “collaborative” AND “argumentation.” The SLR method includes planning, implementation, and reporting. The initial search yielded 877 articles. The researcher applied filtering rules such as articles published between 2009 and 2024, articles published in journals, and research related to collaborative argumentation conducted in the field of mathematics education. The researcher identified 8 main articles that were analyzed according to the research objectives. The analysis results align with the research objectives. The researcher identified the definition of collaborative argumentation, situations requiring collaborative argumentation, characteristics of tasks used in collaborative argumentation studies, and frameworks or approaches used to analyze collaborative argumentation in mathematics education research. Collaborative argumentation is a dialog between two or more people when they work together to determine a solution to a problem they are facing. One situation that requires collaborative argumentation involves forming students into several groups so that they can work together to complete the problem-solving tasks given. The tasks used in collaborative argumentation studies are mostly nonroutine tasks that require students to collaborate with their peers. The types of frameworks used by researchers to analyze collaborative argumentation include the Toulmin argument pattern, collaborative creative reasoning, the Toulmin model supplemented with Habernas' rationality construct, the zigzag pathway in geometric reasoning accompanied by consideration of attitude factors in problem solving, teachers' understanding when involved in argumentation activities in the classroom, emotions when students solve problems, mathematical reasoning supplemented with semiotic indicators, and semantic warrant analysis in substantial argumentation activities. The findings of the SLR can provide a strong theoretical and methodological foundation for further research.

Keywords: argument, student discourse, dialog, frameworks, problem-solving task

1. Introduction

An argument is a set of statements (propositions) consisting of three parts: a conclusion, a set of premises, and a conclusion. Other arguments can support an argument or be criticized through critical questioning of the argument (Walton, 2009). Arguments often occur in aspects of our daily lives. We usually argue with friends over trivial matters such as which restaurant to visit, with a doctor, how to determine the best therapy for an illness, or with some anonymous users on social networks (Baroni et al., 2020). claim, data, warrant, and backing (Evagorou et al., 2023). Argument and argumentation are often considered the same, even though they have different meanings. Argumentation is a dialog between two or more people (Erduran et al., 2004).

An argument can be defined as a series of written claims, data, warrants, and backings that contribute to the content of an argument. In the Toulmin argument pattern (TAP), the essential elements are as follows: The word argumentation has a connotation of being aggressive, contentious, unpleasant, and quarrelsome. However, argumentation is a way for individuals to justify their beliefs and behaviors to influence each other (Willmot, 2000). Toulmin (2003) defines argumentation as the process by which a person convinces others of his claims. According to Krummheuer (2000), argumentation in mathematics education is defined as the interaction that occurs in the classroom and is observed during or after a deliberate explanation of the reasons underlying a solution. In addition, Conner defines argumentation as a process in which more than one person (student/teacher) makes mathematical claims and provides evidence to support the given claims (Conner et al., 2014; Taşdan et al., 2024)

Collaborative argumentation is the topic be discussed in this study. Collaborative argumentation is a dialogue between two or more people, usually exchanging statements and questions, making claims with supporting reasons and evidence, and



critically questioning each other's arguments. This can result in agreement or disagreement with one another (O'Donnell et al., 2011). Collaborative argumentation is the interaction between individuals working together to build arguments and can be beneficial for developing argumentation skills for the students involved (Zheng et al., 2023)

There are many different studies on collaborative argumentation. Therefore, the researcher needs to collect and understand the research results related to collaborative argumentation in mathematics education through a systematic review of the literature about collaborative argumentation over the past 15 years. This paper addresses four research questions. 1) What is collaborative argumentation? 2) What situations demand collaborative argumentation? 3) What characteristics of mathematical tasks can trigger collaborative argumentation? 4) Fourth, what is the framework used to analyze collaborative argumentation?

2. Materials and Methods

A systematic literature review (SR) synthesizes scientific evidence to answer a research question transparently, including all published evidence on the topic of interest (Lame & Guillaume, 2019). Specifically, it adapts guidelines (Dodd et al., 2017). The search keyword was "collaborative AND argumentation" (877 articles). A total of 877 article titles were entered into the VoSviewer application to visualize of the research clusters. An overlay visualization of the studies was also created to show the research from 2009 to 2024. The 877 article titles were filtered to include all book titles, theses, proceedings, non-research articles, or those that did not use the keywords collaborative AND argumentation. During abstract reading, the focus was to include articles that specifically investigated collaborative argumentation in mathematical contexts and exclude articles that fell outside this focus. In this stage, eight main articles met these requirements. The complete criteria are presented in Table 1 below.

Table 1 Inclusion and exclusion criteria.

Inclusion Criteria	Exclude Criteria
Articles from 2009-2024	Articles before 2009
Articles published in journals	Articles not written in English
Research conducted in mathematics education	Article published in proceedings
Focus on the keywords Collaborative and Argumentation	Literature Review
	Other contexts

The guidelines have three stages: planning, conducting, and reporting, as shown in Figure 1.

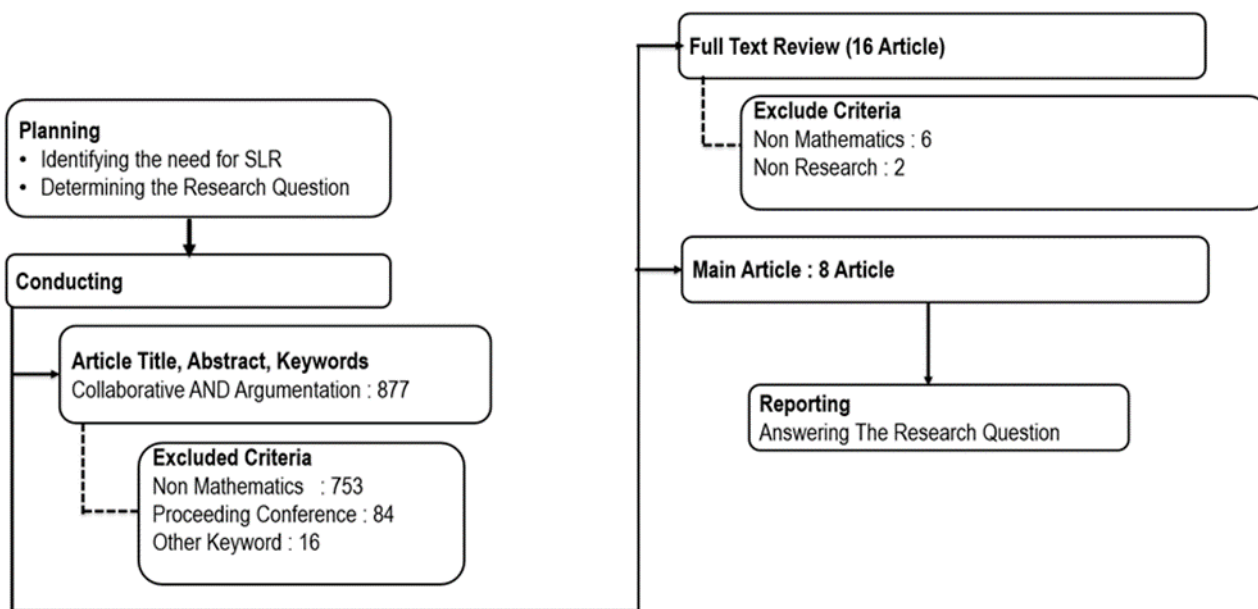


Figure 1 Procedure of the systematic literature review.

3. Results and Discussion

Before answering the research questions, we used VoSviewer to determine the overall trend of studies on collaborative argumentation in mathematics education. We found 877 documents from 2009 to 2024. Through VoSviewer, we identified three clusters. The first cluster in red consists of argumentation skills, impact, collaborative learning, science, development, practice, and scientific argumentation. The second cluster in blue consists of influence, reasoning, case, and web. The third



two or more people to exchange statements, questions, or answers. On this occasion, the participants make claims and support them for various reasons. The dialog in the group aims to resolve disagreements in the discussion (Hmelo-Silver et al., 2013). This provides more interaction with peers, especially when they find that there are different ideas among them. These various ideas can make them curious and help them determine which idea is better (Bahari et al., 2021). The systematic literature review process conducted by researchers on eight articles defines collaborative argumentation as two or more individuals in a group working together to solve a problem or situation that occurred in the learning process when the teacher asked a question. All the students exchanged ideas to determine a solution.

Research conducted by Urhan and Zengin (2024) analyzed the collaborative argumentation of students into several groups to prove the trigonometric limit values. Similarly, by Van Ness and Maher (2019), analyzed the collaborative argumentation of fourth-grade elementary school students during mathematics learning to determine the solution to the fraction comparison problem. The exact process also occurs in research by Kazemi et al. (2021), which reveals the importance of teacher arguments so that collaborative argumentation can arise when determining the truth/falsity of a mathematical statement related to fraction and multiplication material. Gómez-Chacón et al. (2016) analyzed the argument of a group of students in solving GeoGebra-assisted geometry problems. Ayalon et al. (2022) analyzed emotions through the dialogue of a group of students consisting of 3 boys and three girls in solving geometry and measurement problems. Carlsen (2018) analyzed mathematical reasoning through high school students who were divided into five groups. Each group collaborated in determining the equation of the graph of a sinusoidal function. Granberg and Olsson (2015) analyzed creative reasoning through collaborative argumentation among 18 students, and each group worked together to solve linear function problems with GeoGebra. Walter and Barros (2011) analyzed the substantial argumentation of four groups of students, who determined the volume of objects using integrals.

These eight articles describe the research process of collaborative argumentation. Students in groups collaborate in solving the problems faced. When learning takes place, the teacher gives a mathematical problem, and the class determines the solution together. This provides a different view of how research on collaborative argumentation can be done appropriately.

3.2. Characteristics of math tasks

Mathematics tasks are classroom activities that aim to focus students' attention on specific mathematical ideas (Stein et al., 1996). There are several types of math tasks (Hidayah & Forgasz, 2020; Zhu & Fan, 2006).

- Tasks in a purely mathematical form, function in a verbal form, tasks in a visual form, and tasks in a combined form. If the task presentation includes only mathematical expressions, then the task is categorized as a purely mathematical task. If the task presentation only uses written words, then the task is classified as a verbal task. If the task presentation consists of pictures, graphs, figures, and so on, then the task is categorized as a visually. If the task presentation combines two or three of the above, it is classified as a combined task.

- Routine tasks and nonroutine tasks.

Routine tasks can be completed via known procedures. On the other hand, nonroutine tasks are tasks that cannot be solved via previously known formulas, procedures, or logarithms.

- Application problems vs. non-application Problems

Actual context tasks are related to everyday contexts. On the other hand, noncontextual tasks have nothing to do with everyday life situations. Examples of actual context tasks are games or puzzles.

- Open-ended problems and closed-ended problems.

There are two definitions of open-ended tasks: 1) tasks that demand multiple solutions (Kwon et al., 2006) and 2) mathematical tasks that a variety of procedures can solve (Sanchez, 2013). Closed-ended problems are problems that have only one way to obtain an answer (Stein et al., 1996).

Table 2 below explains the characteristics of the tasks from the eight reviewed articles.

The researcher also analyzed eight key articles related to the research topic and the educational level of the research participants, which are presented in the table 3 below.

3.3. Framework for analyzing collaborative argumentation

Generally, a framework is a tangible or conceptual structure intended to support or guide the construction of something that extends the structure into something useful (Lutkevich, 2020). The eight main articles have different frameworks for analyzing collaborative argumentation tailored to the research objectives. Urhan and Zengin (2024) used the Toulmin model framework, complemented by Habermas' rationality constructs, to analyze argumentation during the proof process. The Toulmin model, complemented by Habermas' rationality constructs, highlights the three aspects contained in the warrant: epistemic, teleological, and communicative rationality, as shown in Figure 4.

Table 2 Characteristics Of math tasks for collaborative argumentation.

Study	Kinds of Task									
	Purely Mathematics Form	Verbal Form	Visual Form	Combined Form	Routine task	nonroutine task.	Application Problems	Non application Problems	Open Ended	Close Ended
(Urhan & Zengin, 2024)		✓				✓		✓	✓	
(Van Ness & Maher, 2019)			✓			✓	✓		✓	
(Kazemi et al., 2021)	✓				✓			✓	✓	
(Gómez-Chacón et al., 2016)		✓				✓		✓	✓	
(Ayalon et al., 2022)		✓				✓	✓		✓	
(Carlsen, 2018)			✓			✓		✓		✓
(Granberg & Olsson, 2015)		✓				✓		✓	✓	
(Walter & Barros, 2011)		✓				✓	✓		✓	

Table 3 Research topic and participant education level.

Author	Research Topic	Participant Education Level
(Urhan & Zengin, 2024)	Limit	University
(Van Ness & Maher, 2019)	Fraction	Elementary School
(Kazemi et al., 2021)	Fractions and Multiplication	Elementary School
(Gómez-Chacón et al., 2016)	Fractions and Multiplication	Junior High School
(Ayalon et al., 2022)	Function	Junior High School
(Carlsen, 2018)	Trigonometry	Senior High School
(Granberg & Olsson, 2015)	Function	Senior High School
(Walter & Barros, 2011)	Integration	University

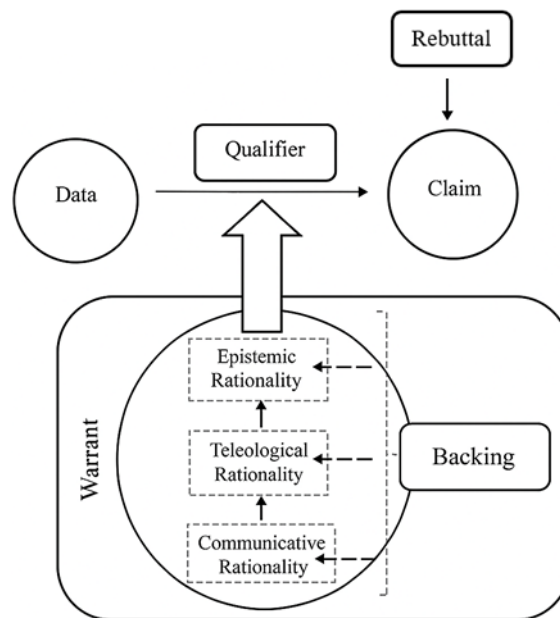


Figure 4 Integration of Habermas' construct of rationality and Toulmin's model (the raationality components are provided in the dotted boxes).

Source: Urhan & Zengin (2024).

Kazemi et al. (2021) research aims to understand teachers' understanding of argumentation in the classroom when they are involved in professional development (PD) design. The professional development design was conducted in two different elementary schools. There are three stages in the data analysis process. First, the researcher recorded all activities in the Learning Lab. Second, by understanding the definition of sensemaking, Kazemi created a more detailed report, organized in



tables, with low inferential descriptive text, direct quotes, and images from the field notes. Third, the original video and audio recordings will be reviewed to check accuracy and add detail to the field notes.

Gómez-Chacón et al. (2016) aimed to describe the interaction between cognition and affect in the transition from instrumental to discursive genesis in geometric reasoning. The dialogue of group participants when solving problems was analyzed via the zig-zag pathway in geometric reasoning. This zig-zag phenomenon that the ideas in solving the problem do not move up or down but rather follow a zig-zag pathway. The idea, initially considered a temporary conjecture, progresses to the following conjecture and is revisited to be replaced by a theorem. This process takes place via a zig-zag pathway in geometric reasoning. In addition, when analyzing students' argumentation data, Gómez-Chacón also considered affective factors such as satisfaction in doing mathematics, confidence, perseverance, and independence.

Van Ness and Maher (2019) conducted their research via a diagrammatic framework inspired by the work of Krummheuer (1995, 1997, 2000, 2007, 2013) and others. Each diagram shows the data, claims, warrants, and backings presented by the students during the argumentation session. The framework is shown in Figure 5.

Ayalon et al. (2022) research aims to analyze students' emotions while solving mathematical problems together. Ayalon used 12 of the 13 emotion indicators from Else-Quest et al. (2008). Each emotion category is described in Table 4.

Carlsen (2018) conducted data analysis with a dialogical approach of three principles (Linell, 1998). First, there is sequentiality. This principle describes how the position of students is involved in the discussion. Second, coconstruction means that the dialogue in each group is a form of cooperation in determining the equation of a sinusoidal function. Third, interdependence means that students depend on each other to collaborate in problem-solving. The dialogical approach used to describe mathematical reasoning is complemented by semiotics, as well as gestures, voice, mathematical language related to sinusoidal functions, deictics, notebooks, and lecture notes.

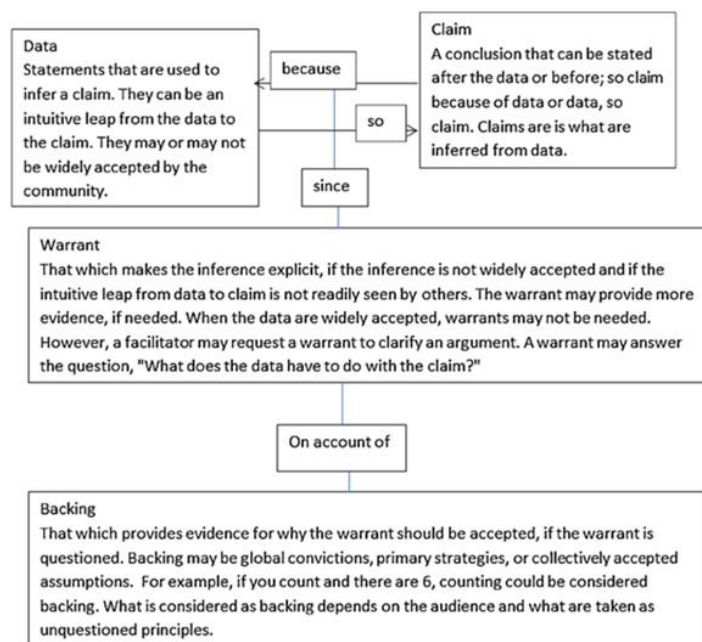


Figure 5 Prototype diagram for mapping argumentation in the discussion. Source: Van Ness & Maher (2019).

Table 4 Indicators of emotions.

CATEGORY	DESCRIPTION
Tension	Nervousness, anxiety, uncertainty, self-consciousness, worry, tension
Distress/Dismay	Distress, complaining, disappointment
Frustration	Frustration, annoyance, impatience
Sadness	Sadness, withdrawal, self-criticism
Boredom/Apathy	Boredom, apathy, absence of all interest, fat
Contempt	Contempt, mocking, sarcasm, smugness, disrespect, scorn, "brattiness"
Positive Interest	Interest, engagement, positive attention, eagerness
Affection/Caring	Encouragement, support, soothing, reassurance, trust, respect, warmth
Joy/Pleasure	Happiness, excitement, pleased, having fun, enjoyment
Humor	Humor, joking, friendly teasing, silliness
Pride	Pride, amazement, focused on achievement or ability
Off Task	The group is not working on the mathematics task



Granberg and Olsson (2015) to answer the research questions, the data were organized and analyzed via the creative reasoning framework described by Lithner (2008) and collaboratively by Roschelle & Teasley (1995). The analysis focused on students' language (suggestions, questions, answers, arguments, etc.) and actions (movements and interactions during the use of GeoGebra in problem-solving). Collaboration is "coordinated and synchronized activities that result from sustained efforts to build and maintain a shared conception of a problem. The research framework described by Lithner (2008) defines a student's reasoning as his or her train of thought guided and constrained by the student's competencies created within a sociocultural environment. The student's thought process, as an invisible act of the mind, can be articulated and, thus, traced in the form of an observable reasoning sequence. The reasoning sequence starts with a task, consists of all actions, including completing the task, and ends with a correct or incorrect conclusion or a decision to give up. The following is a flow of creative reasoning for individuals, as presented in the Figure 6 below.

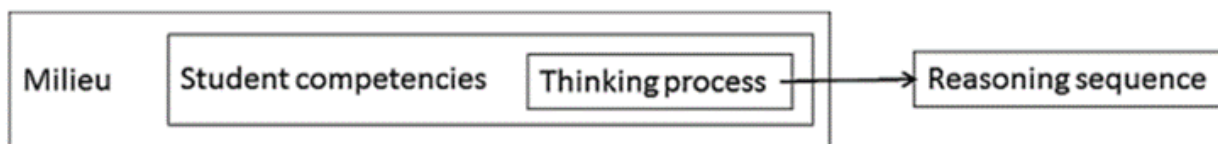


Figure 6 Framework for the reasoning sequence. Source: Lithner (2008).

The didactical design of this study includes students working in pairs. Since the framework presented by Lithner (2008) addresses individual reasoning, the collaborative perspective when solving problems with the help of GeoGebra becomes the flow shown in Figure 7

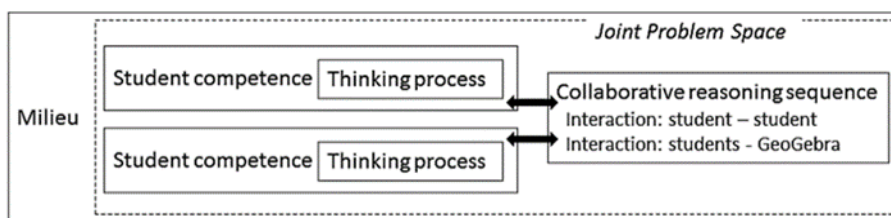


Figure 7 Framework for analysing the collaborative reasoning sequence. Source: Granberg & Olsson (2015).

Roschelle & Teasley (1995) introduced the joint problem space as a jointly negotiated and socially negotiated knowledge structure consisting of a goal, a description of the current state of the problem, and an awareness of available problem-solving actions. A joint problem space allows students to confer or discuss solving problems and reach a mutual agreement. The joint problem space will enable students to use their ideas to contribute to achieving a common goal, shared knowledge, and steps to solve a problem. Students who join a group must observe the problem-solving process and correct misunderstandings until an agreement is reached to decide how to solve the problem.

Walter and Barros (2011) research aims to determine the substantial argument of students to convince themselves and others of the truth of mathematics. Specifically, how do students work together to develop and guarantee a mathematical method to find the volume of a solid when no prior instruction on using the method or solution is given? The framework used was a microlinguistic analysis of the students' dialogs, which included open and axial coding to examine the structure of the students' substantial argumentation and support the identification and characterization of the semantic warrant that the students proposed. A semantic warrant is a meaningful example that underlies reasoning and supports mathematical conclusions. Students submit semantic warrants for peer consideration or approval in the learning community's social environment. Students' submission of semantic warrants is recognized as an attempt to convince themselves and others of the truth of their personally constructed mathematics. Open coding involved repeated playbacks of the videos and transcripts to capture students' use of language and notation that indicated students' attempts at justification, argumentation, consensus, and proof. Axial coding aims to reveal the existence of mathematical references in the form of personal initiatives that can be used as data or support in substantial argumentation to strengthen the level of warrant validity further.

Argumentation is a type of discourse that involves individuals or groups working together to evaluate claims of knowledge that have been constructed to include evidence, either theoretical or empirical (Erduran et al., 2015). Argumentation is a verbal, social, and rational activity that aims to convince others who do not accept a point of view by presenting a series of statements that seek to justify or refute the statements expressed (Sampson et al., 2013). A characteristic of argumentation is that there is a response individual who shows acceptance of the defended point of view. This means that argumentation is not just a monologue; argumentation is, in principle, part of a dialogue, so it is not only a complex act of communication with the aim of understanding but also an interactional act to achieve acceptance. When argumentation is presented in a complete discussion, the dialogue is explicit. The dialog will remain implicit when argumentation is addressed to a noninteractive listener or reader. Another common characteristic of argumentation is that it is a rational activity of reason so that the arguer can be held accountable (Eemeren, 2018).



Collaborative argumentation is a dialogue between two or more people, usually exchanging statements and questions, making claims with supporting reasons and evidence, and critically questioning each other's arguments. This can result in agreement or disagreement with each other (Hmelo-Silver et al., 2013). Collaborative argumentation starts with a disagreement, so individuals try to harmonize their ideas and perceptions. One of the situations that creates collaborative argumentation in mathematics education is when students discuss with each other to solve a problem. We already know that one of the conditions for collaborative argumentation is when students work together to prove the value of trigonometric limits using GeoGebra, and there are also several conditions when classroom learning is carried out. Based on these findings, collaborative argumentation can be built from learning activities with materials that trigger argumentation or in collaborative problem-solving situations. According to OECD (2017), in the social aspect of CPS, there is a need for two or more individuals to communicate, exchange ideas, and identify the problem at hand, such as identifying what is known, identifying the problem to be solved, making a problem-solving plan, and reevaluating the solution that has been obtained. In CPS activities, arguments from each individual are certainly involved when problem-solving occurs. This triggers a dialog between two or more people when solving crypt-arithmetic problems. Thus, CPS is the background of this research. This is a reference for researchers to discover the situations that trigger collaborative argumentation in mathematics education.

The educational levels of the participants are diverse, ranging from elementary school to college. The types of math problem-solving tasks given at each educational level are different. This can be used as a separate reference if research related to collaborative argumentation is carried out in the context of mathematics education. The framework used to analyze collaborative argumentation varies according to the research objectives. Some analyses use the Toulmin argument pattern when students in elementary school discuss determining fraction comparisons (Van Ness & Maher, 2019). Urhan and Zengin (2024) use the Toulmin Model framework, which is complemented by the Habermas rationality construct, to analyze argumentation during the proof process. The Toulmin model, which is complemented by the Habermas rationality construct, highlights the three aspects contained in the warrant: epistemic rationality, teleological rationality, and communicative rationality. Kazemi et al. (2021) analyzed the teachers' understanding of their his involvement in the argumentation process in the classroom. Gómez-Chacón et al. (2016) use the zig-zag pathway in geometric reasoning.

Ayalon et al. (2022) used 12 of 13 emotion indicators from Else-Quest et al. (2008) to determine students' emotions during collaboration in problem-solving. Carlsen (2018) uses a dialogical approach to analyze mathematical reasoning with a semiotic approach, and Walter and Barros (2011) use the TAP to determine the semantic warrant in solving solid volume problems. Some frameworks used above allow researchers to analyze collaborative argumentation when students engage in problem-solving activities. Thus, problem-solving is found at levels of education (Susanti et al., 2023). Toulmin Argument Pattern (TAP) is the most frequently used framework in analyzing collaborative argumentation. We can find that three studies have used the Toulmin argument pattern to explore collaborative argumentation (Toulmin, 2003). Through TAP, we can describe collaborative argumentation and identify its patterns. Argumentation patterns can represent the claim determination process and shed light on how individuals engage in argumentation. As such, argumentation patterns can be used to build psychological theories about how individuals understand, generate, and evaluate arguments. Therefore, if researchers want to analyze the collaborative argumentation process, using the Toumin argument pattern is more appropriate.

5. Final Considerations

The results of the systematic literature review of eight main articles explained the definition of collaborative argumentation, the situations that demand collaborative argumentation, the characteristics of mathematical tasks for collaborative argumentation, and the framework used to analyze the dialogue when collaborative argumentation takes place in solving problems. Collaborative argumentation is an interaction between two or more individuals working together to determine a claim with supporting evidence and reasons. Through collaborative argumentation, students can exchange ideas and criticize each other's ideas between peers, which leads to an agreement in determining a claim. The situation that requires collaborative argumentation is when of students work in groups to solve a mathematical problem. The characteristics of the problem-solving tasks used to generate collaborative argumentation situations are mostly open-ended and non-routine tasks. This is a unique characteristic of the mathematical tasks given to students that allows for collaborative argumentation situations. The framework used to analyze students' dialogue in collaborative argumentation also varies greatly. Based on the results of the SLR of eight articles, the TAP framework, collaborative creative reasoning, the Toulmin model equipped with Habermas rationality constructs, the zig-zag pathway in geometry reasoning accompanied by consideration of attitudinal factors in solving problems, teacher understanding when involved in argumentation activities in the classroom, emotions, mathematical reasoning equipped with semiotic indicators, and semantic warrant analysis on substantial argumentation activities. Although much research has been conducted on collaborative argumentation in the context of mathematics education, there are still significant gaps in the research that has been undertaken, such as how researchers analyze the participation of each individual involved in collaborative argumentation rather than relying solely on the Toulmin argument pattern framework, which is the majority used for analyzing collaborative argumentation. Through participation analysis, we can identify the types of individuals involved in the argument who are responsible for the statements made, who steal their

friends' ideas, or who repeat their friends' statements. This has attracted researchers to conduct further research on collaborative argumentation.

Ethical Considerations

Not applicable.

Conflict of Interest

The authors declare no conflicts of interest.

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